

Online Resource toTitle: **On uniformly resolvable designs with block sizes 3 and 4**

Journal: Designs, Codes and Cryptography

Ernst Schuster

*Institute for Medical Informatics, Statistics and Epidemiology, University of Leipzig,
Härtelstr. 16/18, 04107 Leipzig, Germany*e-mail: Ernst.Schuster@imise.uni-leipzig.de

Gennian Ge

Department of Mathematics, Zhejiang University, Hangzhou 310027, Zhejiang, P. R. China

This Online Resource contains ingredient designs required for our constructions. These were found computationally.

Example A.1 An LRPBD₄(3; 6) ; each row forms a parallel class:

(1 5 6; 0 1 1), (2 3 4; 2 2 0),
 (2 3 6; 2 2 0), (1 4 5; 0 2 2),
 (2 4 5; 2 0 2), (1 3 6; 1 0 3),
 (3 4 5; 1 2 1), (1 2 6; 3 0 1),
 (1 3 5; 3 2 3), (2 4 6; 2 3 1),
 (1 4 6; 3 3 0), (2 3 5; 3 0 1),
 (4 5 6; 1 2 1), (1 2 3; 1 0 3),
 (1 2 5; 2 3 1), (3 4 6; 2 3 1),
 (3 5 6; 0 0 0), (1 2 4; 1 0 3),
 (2 5 6; 3 3 0), (1 3 4; 2 3 1).

Example A.2 An LRTD₄(4, 2) , $\mathbf{G} = \{\{1,2\}, \{3,4\}, \{5,6\}, \{7,8\}\}$; each row forms a parallel class:

(1 4 5 8; 0 1 2 1 2 1), (2 3 6 7; 1 0 2 3 1 2),
 (1 3 5 7; 3 3 2 0 3 3), (2 4 6 8; 2 2 2 0 0 0),
 (2 3 5 8; 0 1 1 1 1 0), (1 4 6 7; 3 0 1 1 2 1),
 (1 4 5 7; 1 0 0 3 3 0), (2 3 6 8; 2 3 0 1 2 1),
 (2 4 5 8; 0 0 3 0 3 3), (1 3 6 7; 1 3 3 2 2 0),
 (2 4 6 7; 3 1 0 2 1 3), (1 3 5 8; 0 2 0 2 0 2),
 (1 4 6 8; 2 1 3 3 1 2), (2 3 5 7; 3 2 3 3 0 1),
 (1 3 6 8; 2 2 1 0 3 3), (2 4 5 7; 1 3 1 2 0 2).

Removing the points 7 and 8 with there labels gives an LRTD₄(3, 2) .

Example A.3 An LURD₃(\{3, 4\}; 24) with $r_4 = 9$; each row forms a uniform parallel class:

(1 2 3; 2 2 0), (4 5 6; 0 2 2), (7 8 9; 0 2 2), (10 11 12; 0 2 2), (13 14 15; 1 1 0), (16 17 18; 2 1 2), (19 20 21; 2 1 2), (22 23 24; 2 1 2),
 (1 17 23; 1 2 1), (2 18 24; 0 0 0), (3 19 21; 0 2 2), (4 20 22; 1 0 2), (5 9 13; 0 0 0), (6 10 14; 0 0 0), (7 11 15; 0 1 1), (8 12 16; 0 0 0),
 (5 11 21; 0 2 2), (6 12 22; 2 2 0), (7 9 23; 0 0 0), (8 10 24; 0 1 1), (1 16 18; 2 2 0), (2 13 19; 1 0 2), (3 14 20; 2 0 1), (4 15 17; 0 1 1),
 (5 14 17; 1 0 2), (6 15 18; 0 2 2), (7 16 19; 2 0 1), (8 13 20; 1 2 1), (1 9 21; 0 0 0), (2 10 22; 2 0 1), (3 11 23; 0 1 1), (4 12 24; 2 1 2),
 (9 14 18; 2 0 1), (10 15 19; 0 1 1), (11 16 20; 0 2 2), (12 13 17; 2 2 0), (1 5 22; 0 0 0), (2 6 23; 0 0 0), (3 7 24; 2 1 2), (4 8 21; 0 0 0),
 (1 10 20; 0 2 2), (2 11 17; 0 1 1), (3 12 18; 0 1 1), (4 9 19; 1 0 2), (5 16 23; 2 2 0), (6 13 24; 0 1 1), (7 14 21; 0 0 0), (8 15 22; 1 0 2),
 (1 15 24; 0 0 0), (2 16 21; 0 1 1), (3 13 22; 1 0 2), (4 14 23; 0 2 2), (5 12 20; 1 0 2), (6 9 17; 2 1 2), (7 10 18; 0 1 1), (8 11 19; 0 1 1),
 (13 18 21; 2 1 2), (14 19 22; 1 0 2), (15 20 23; 1 2 1), (16 17 24; 0 0 0), (1 7 12; 1 0 2), (2 8 9; 1 2 1), (3 5 10; 1 2 1), (4 6 11; 1 1 0),
 (1 6 19; 1 1 0), (2 7 20; 2 1 2), (3 8 17; 0 0 0), (4 5 18; 1 2 1), (9 16 22; 1 0 2), (10 13 23; 0 1 1), (11 14 24; 1 0 2), (12 15 21; 0 1 1),
 (9 20 24; 2 2 0), (10 17 21; 2 0 1), (11 18 22; 1 2 1), (12 19 23; 2 2 0), (1 8 14; 1 2 1), (2 5 15; 0 0 0), (3 6 16; 2 0 1), (4 7 13; 1 1 0),
 (5 19 24; 1 0 2), (6 20 21; 2 0 1), (7 17 22; 0 2 2), (8 18 23; 0 2 2), (1 11 13; 0 0 0), (2 12 14; 2 0 1), (3 9 15; 1 1 0), (4 10 16; 2 2 0),
 (1 3 19; 1 0 2), (2 4 20; 0 0 0), (5 21 23; 0 1 1), (6 22 24; 0 2 2), (7 11 15; 1 0 2), (8 12 16; 1 2 1), (9 13 17; 2 1 2), (10 14 18; 2 2 0),

$(1\ 7\ 10; 0\ 1\ 1), (3\ 6\ 9; 1\ 1\ 0), (2\ 5\ 12; 1\ 1\ 0), (4\ 8\ 11; 0\ 0\ 0),$
 $(5\ 9\ 10; 0\ 0\ 0), (2\ 4\ 7; 1\ 1\ 0), (1\ 6\ 11; 0\ 1\ 1), (3\ 8\ 12; 1\ 0\ 1),$
 $(2\ 9\ 11; 0\ 1\ 1), (1\ 5\ 8; 1\ 0\ 1), (6\ 7\ 12; 0\ 0\ 0), (3\ 4\ 10; 1\ 1\ 0),$
 $(1\ 4\ 9\ 11; 1\ 0\ 0\ 1\ 1\ 0), (2\ 6\ 8\ 12; 1\ 0\ 0\ 1\ 1\ 0), (3\ 5\ 7\ 10; 1\ 0\ 0\ 1\ 1\ 0),$
 $(2\ 5\ 7\ 11; 0\ 0\ 0\ 0\ 0\ 0), (3\ 4\ 9\ 12; 0\ 0\ 1\ 0\ 1\ 1), (1\ 6\ 8\ 10; 1\ 1\ 0\ 0\ 1\ 1),$
 $(2\ 6\ 9\ 10; 0\ 1\ 0\ 1\ 0\ 1), (1\ 4\ 7\ 12; 0\ 1\ 0\ 1\ 0\ 1), (3\ 5\ 8\ 11; 0\ 0\ 1\ 0\ 1\ 1),$
 $(1\ 5\ 9\ 12; 0\ 1\ 1\ 1\ 1\ 0), (2\ 4\ 8\ 10; 0\ 1\ 1\ 1\ 1\ 0), (3\ 6\ 7\ 11; 0\ 1\ 0\ 1\ 0\ 1).$

Below some designs, which are constructed with help of difference families. These are found computationally.

Example A.6 There exists a URD($\{3, 4\}; 60$) with $r_4 = 7$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on

$X = Z_4 \times Z_{15}$. Take the following seven parallel classes with blocks of size 4:

$$\begin{aligned} P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-15} \\ P_2 &= \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-15} \\ P_3 &= \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-15} \\ P_4 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-15} \\ P_5 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-15} \\ P_6 &= \{(0,0), (1,3), (2,6), (3,10)\} \pmod{-15} \\ P_7 &= \{(0,10), (1,6), (2,3), (3,0)\} \pmod{-15} \end{aligned}$$

It is well known that there is an RPBD(3; 15) with seven parallel classes. Place a copy of this design on each Z_{15} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{15} the parallel class is placed and $j = 1, \dots, 7$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{array}{ll} \{(0,0),(1,8),(2,1)\} \pmod{-15} \cup R_{3,1} & \{(0,0),(1,10),(3,13)\} \pmod{-15} \cup R_{2,1} \\ \{(0,0),(1,9),(2,5)\} \pmod{-15} \cup R_{3,2} & \{(0,0),(1,6),(3,14)\} \pmod{-15} \cup R_{2,2} \\ \{(0,0),(1,7),(2,12)\} \pmod{-15} \cup R_{3,3} & \{(0,0),(1,5),(3,11)\} \pmod{-15} \cup R_{2,3} \\ \{(0,0),(1,4),(2,10)\} \pmod{-15} \cup R_{3,4} & \{(0,0),(1,12),(3,7)\} \pmod{-15} \cup R_{2,4} \\ \\ \{(0,0),(2,14),(3,2)\} \pmod{-15} \cup R_{1,1} & \{(1,0),(2,4),(3,12)\} \pmod{-15} \cup R_{0,1} \\ \{(0,0),(2,7),(3,1)\} \pmod{-15} \cup R_{1,2} & \{(1,0),(2,7),(3,14)\} \pmod{-15} \cup R_{0,2} \\ \{(0,0),(2,9),(3,4)\} \pmod{-15} \cup R_{1,3} & \{(1,0),(2,10),(3,1)\} \pmod{-15} \cup R_{0,3} \\ \{(0,0),(2,3),(3,8)\} \pmod{-15} \cup R_{1,4} & \{(1,0),(2,9),(3,5)\} \pmod{-15} \cup R_{0,4}. \end{array}$$

The last three parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,5}, \bigcup_{i=0}^3 R_{i,6}$ and $\bigcup_{i=0}^3 R_{i,7}$.

Example A.7 There exists a URD($\{3, 4\}; 60$) with $r_4 = 9$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{15}$. Take the following nine parallel classes with blocks of size 4:

$$\begin{aligned}
P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-15} \\
P_2 &= \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-15} \\
P_3 &= \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-15} \\
P_4 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-15} \\
P_5 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-15} \\
P_6 &= \{(0,0), (1,3), (2,6), (3,10)\} \pmod{-15} \\
P_7 &= \{(0,10), (1,7), (2,4), (3,0)\} \pmod{-15} \\
P_8 &= \{(0,0), (1,4), (2,8), (3,13)\} \pmod{-15} \\
P_9 &= \{(0,13), (1,9), (2,5), (3,0)\} \pmod{-15}
\end{aligned}$$

It is well known that there is an RPBD(3; 15) with seven parallel classes. Place a copy of this design on each Z_{15} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{15} the parallel class is placed and $j = 1, \dots, 7$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{array}{ll}
\{(0,0),(1,7),(2,12)\} \pmod{-15} \cup R_{3,1} & \{(0,0),(1,10),(3,7)\} \pmod{-15} \cup R_{2,1} \\
\{(0,0),(1,5),(2,14)\} \pmod{-15} \cup R_{3,2} & \{(0,0),(1,9),(3,14)\} \pmod{-15} \cup R_{2,2} \\
\{(0,0),(1,8),(2,1)\} \pmod{-15} \cup R_{3,3} & \{(0,0),(1,6),(3,1)\} \pmod{-15} \cup R_{2,3} \\
\\
\{(0,0),(2,10),(3,4)\} \pmod{-15} \cup R_{1,1} & \{(1,0),(2,7),(3,14)\} \pmod{-15} \cup R_{0,1} \\
\{(0,0),(2,5),(3,8)\} \pmod{-15} \cup R_{1,2} & \{(1,0),(2,6),(3,3)\} \pmod{-15} \cup R_{0,2} \\
\{(0,0),(2,3),(3,11)\} \pmod{-15} \cup R_{1,3} & \{(1,0),(2,10),(3,1)\} \pmod{-15} \cup R_{0,3}.
\end{array}$$

The last four parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,4}$, $\bigcup_{i=0}^3 R_{i,5}$, $\bigcup_{i=0}^3 R_{i,6}$ and $\bigcup_{i=0}^3 R_{i,7}$.

Example A.8 There exists a URD($\{3, 4\}; 132$) with $r_4 = 7$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{33}$. Take the following seven parallel classes with blocks of size 4:

$$\begin{aligned}
P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-33} \\
P_2 &= \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-33} \\
P_3 &= \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-33} \\
P_4 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-33} \\
P_5 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-33} \\
P_6 &= \{(0,0), (1,3), (2,6), (3,9)\} \pmod{-33} \\
P_7 &= \{(0,9), (1,6), (2,3), (3,0)\} \pmod{-33}
\end{aligned}$$

It is well known that there exists an RPBD(3; 33) with 16 parallel classes. Place a copy of this design on each Z_{33} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{33} the parallel class is placed and $j = 1, \dots, 16$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{aligned}
& \{(0,0),(1,5),(2,30)\} \pmod{-33} \cup R_{3,1} & & \{(0,0),(1,6),(3,29)\} \pmod{-33} \cup R_{2,1} \\
& \{(0,0),(1,22),(2,32)\} \pmod{-33} \cup R_{3,2} & & \{(0,0),(1,23),(3,20)\} \pmod{-33} \cup R_{2,2} \\
& \{(0,0),(1,27),(2,12)\} \pmod{-33} \cup R_{3,3} & & \{(0,0),(1,26),(3,18)\} \pmod{-33} \cup R_{2,3} \\
& \{(0,0),(1,11),(2,18)\} \pmod{-33} \cup R_{3,4} & & \{(0,0),(1,12),(3,17)\} \pmod{-33} \cup R_{2,4} \\
& \{(0,0),(1,25),(2,3)\} \pmod{-33} \cup R_{3,5} & & \{(0,0),(1,16),(3,4)\} \pmod{-33} \cup R_{2,5} \\
& \{(0,0),(1,15),(2,10)\} \pmod{-33} \cup R_{3,6} & & \{(0,0),(1,19),(3,31)\} \pmod{-33} \cup R_{2,6} \\
& \{(0,0),(1,13),(2,22)\} \pmod{-33} \cup R_{3,7} & & \{(0,0),(1,28),(3,15)\} \pmod{-33} \cup R_{2,7} \\
& \{(0,0),(1,20),(2,26)\} \pmod{-33} \cup R_{3,8} & & \{(0,0),(1,29),(3,11)\} \pmod{-33} \cup R_{2,8} \\
& \{(0,0),(1,18),(2,1)\} \pmod{-33} \cup R_{3,9} & & \{(0,0),(1,24),(3,13)\} \pmod{-33} \cup R_{2,9} \\
& \{(0,0),(1,7),(2,24)\} \pmod{-33} \cup R_{3,10} & & \{(0,0),(1,9),(3,23)\} \pmod{-33} \cup R_{2,10} \\
& \{(0,0),(1,8),(2,23)\} \pmod{-33} \cup R_{3,11} & & \{(0,0),(1,14),(3,7)\} \pmod{-33} \cup R_{2,11} \\
& \{(0,0),(1,21),(2,14)\} \pmod{-33} \cup R_{3,12} & & \{(0,0),(1,4),(3,28)\} \pmod{-33} \cup R_{2,12} \\
& \{(0,0),(1,17),(2,5)\} \pmod{-33} \cup R_{3,13} & & \{(0,0),(1,10),(3,26)\} \pmod{-33} \cup R_{2,13} \\
\\
& \{(0,0),(2,28),(3,10)\} \pmod{-33} \cup R_{1,1} & & \{(1,0),(2,22),(3,13)\} \pmod{-33} \cup R_{0,1} \\
& \{(0,0),(2,9),(3,25)\} \pmod{-33} \cup R_{1,2} & & \{(1,0),(2,23),(3,3)\} \pmod{-33} \cup R_{0,2} \\
& \{(0,0),(2,19),(3,12)\} \pmod{-33} \cup R_{1,3} & & \{(1,0),(2,8),(3,17)\} \pmod{-33} \cup R_{0,3} \\
& \{(0,0),(2,20),(3,16)\} \pmod{-33} \cup R_{1,4} & & \{(1,0),(2,27),(3,11)\} \pmod{-33} \cup R_{0,4} \\
& \{(0,0),(2,11),(3,5)\} \pmod{-33} \cup R_{1,5} & & \{(1,0),(2,12),(3,7)\} \pmod{-33} \cup R_{0,5} \\
& \{(0,0),(2,7),(3,19)\} \pmod{-33} \cup R_{1,6} & & \{(1,0),(2,5),(3,10)\} \pmod{-33} \cup R_{0,6} \\
& \{(0,0),(2,21),(3,32)\} \pmod{-33} \cup R_{1,7} & & \{(1,0),(2,20),(3,28)\} \pmod{-33} \cup R_{0,7} \\
& \{(0,0),(2,13),(3,1)\} \pmod{-33} \cup R_{1,8} & & \{(1,0),(2,19),(3,8)\} \pmod{-33} \cup R_{0,8} \\
& \{(0,0),(2,15),(3,22)\} \pmod{-33} \cup R_{1,9} & & \{(1,0),(2,24),(3,9)\} \pmod{-33} \cup R_{0,9} \\
& \{(0,0),(2,25),(3,2)\} \pmod{-33} \cup R_{1,10} & & \{(1,0),(2,29),(3,19)\} \pmod{-33} \cup R_{0,10} \\
& \{(0,0),(2,17),(3,21)\} \pmod{-33} \cup R_{1,11} & & \{(1,0),(2,14),(3,1)\} \pmod{-33} \cup R_{0,11} \\
& \{(0,0),(2,16),(3,8)\} \pmod{-33} \cup R_{1,12} & & \{(1,0),(2,4),(3,18)\} \pmod{-33} \cup R_{0,12} \\
& \{(0,0),(2,8),(3,14)\} \pmod{-33} \cup R_{1,13} & & \{(1,0),(2,13),(3,32)\} \pmod{-33} \cup R_{0,13}.
\end{aligned}$$

The last three parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,14}$, $\bigcup_{i=0}^3 R_{i,15}$ and $\bigcup_{i=0}^3 R_{i,16}$.

Example A.9 There exists a URD($\{3, 4\}; 132$) with $r_4 = 9$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{33}$. Take the following nine parallel classes with blocks of size 4:

$$\begin{aligned}
P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-33} \\
P_2 &= \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-33} \\
P_3 &= \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-33} \\
P_4 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-33} \\
P_5 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-33} \\
P_6 &= \{(0,0), (1,3), (2,6), (3,9)\} \pmod{-33} \\
P_7 &= \{(0,9), (1,6), (2,3), (3,0)\} \pmod{-33} \\
P_8 &= \{(0,0), (1,4), (2,8), (3,12)\} \pmod{-33} \\
P_9 &= \{(0,12), (1,8), (2,4), (3,0)\} \pmod{-33}.
\end{aligned}$$

It is well known that there is an RPBD(3; 33) with 16 parallel classes. Place a copy of this design on each Z_{33} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{33} the parallel class is placed and $j = 1, \dots, 16$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{aligned}
&\{(0,0),(1,5),(2,23)\} \pmod{-33} \cup R_{3,1} \\
&\{(0,0),(1,24),(2,19)\} \pmod{-33} \cup R_{3,2} \\
&\{(0,0),(1,21),(2,15)\} \pmod{-33} \cup R_{3,3} \\
&\{(0,0),(1,17),(2,1)\} \pmod{-33} \cup R_{3,4} \\
&\{(0,0),(1,6),(2,30)\} \pmod{-33} \cup R_{3,5} \\
&\{(0,0),(1,16),(2,28)\} \pmod{-33} \cup R_{3,6} \\
&\{(0,0),(1,9),(2,24)\} \pmod{-33} \cup R_{3,7} \\
&\{(0,0),(1,10),(2,20)\} \pmod{-33} \cup R_{3,8} \\
&\{(0,0),(1,28),(2,9)\} \pmod{-33} \cup R_{3,9} \\
&\{(0,0),(1,20),(2,12)\} \pmod{-33} \cup R_{3,10} \\
&\{(0,0),(1,23),(2,10)\} \pmod{-33} \cup R_{3,11} \\
&\{(0,0),(1,12),(2,21)\} \pmod{-33} \cup R_{3,12} \\
&\{(0,0),(1,15),(3,4)\} \pmod{-33} \cup R_{2,1} \\
&\{(0,0),(1,13),(3,14)\} \pmod{-33} \cup R_{2,2} \\
&\{(0,0),(1,7),(3,26)\} \pmod{-33} \cup R_{2,3} \\
&\{(0,0),(1,26),(3,11)\} \pmod{-33} \cup R_{2,4} \\
&\{(0,0),(1,18),(3,13)\} \pmod{-33} \cup R_{2,5} \\
&\{(0,0),(1,8),(3,17)\} \pmod{-33} \cup R_{2,6} \\
&\{(0,0),(1,25),(3,7)\} \pmod{-33} \cup R_{2,7} \\
&\{(0,0),(1,22),(3,15)\} \pmod{-33} \cup R_{2,8} \\
&\{(0,0),(1,27),(3,8)\} \pmod{-33} \cup R_{2,9} \\
&\{(0,0),(1,14),(3,19)\} \pmod{-33} \cup R_{2,10} \\
&\{(0,0),(1,19),(3,18)\} \pmod{-33} \cup R_{2,11} \\
&\{(0,0),(1,11),(3,1)\} \pmod{-33} \cup R_{2,12}
\end{aligned}$$

$$\begin{aligned}
&\{(0,0),(2,7),(3,22)\} \pmod{-33} \cup R_{1,1} \\
&\{(0,0),(2,17),(3,25)\} \pmod{-33} \cup R_{1,2} \\
&\{(0,0),(2,32),(3,10)\} \pmod{-33} \cup R_{1,3} \\
&\{(0,0),(2,16),(3,32)\} \pmod{-33} \cup R_{1,4} \\
&\{(0,0),(2,26),(3,2)\} \pmod{-33} \cup R_{1,5} \\
&\{(0,0),(2,3),(3,16)\} \pmod{-33} \cup R_{1,6} \\
&\{(0,0),(2,11),(3,29)\} \pmod{-33} \cup R_{1,7} \\
&\{(0,0),(2,5),(3,31)\} \pmod{-33} \cup R_{1,8} \\
&\{(0,0),(2,22),(3,28)\} \pmod{-33} \cup R_{1,9} \\
&\{(0,0),(2,13),(3,20)\} \pmod{-33} \cup R_{1,10} \\
&\{(0,0),(2,18),(3,23)\} \pmod{-33} \cup R_{1,11} \\
&\{(0,0),(2,14),(3,5)\} \pmod{-33} \cup R_{1,12} \\
&\{(1,0),(2,21),(3,10)\} \pmod{-33} \cup R_{0,1} \\
&\{(1,0),(2,8),(3,20)\} \pmod{-33} \cup R_{0,2} \\
&\{(1,0),(2,6),(3,16)\} \pmod{-33} \cup R_{0,3} \\
&\{(1,0),(2,19),(3,7)\} \pmod{-33} \cup R_{0,4} \\
&\{(1,0),(2,22),(3,12)\} \pmod{-33} \cup R_{0,5} \\
&\{(1,0),(2,26),(3,13)\} \pmod{-33} \cup R_{0,6} \\
&\{(1,0),(2,11),(3,3)\} \pmod{-33} \cup R_{0,7} \\
&\{(1,0),(2,5),(3,24)\} \pmod{-33} \cup R_{0,8} \\
&\{(1,0),(2,23),(3,17)\} \pmod{-33} \cup R_{0,9} \\
&\{(1,0),(2,13),(3,30)\} \pmod{-33} \cup R_{0,10} \\
&\{(1,0),(2,16),(3,11)\} \pmod{-33} \cup R_{0,11} \\
&\{(1,0),(2,7),(3,21)\} \pmod{-33} \cup R_{0,12}.
\end{aligned}$$

The last four parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,13}, \bigcup_{i=0}^3 R_{i,14}, \bigcup_{i=0}^3 R_{i,15}$ and $\bigcup_{i=0}^3 R_{i,16}$.

Example A.10 There exists a URD($\{3, 4\}; 156$) with $r_4 = 7$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{39}$. Take the following seven parallel classes with blocks of size 4:

$$\begin{aligned} P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-39} \\ P_2 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-39} \\ P_3 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-39} \\ P_4 &= \{(0,0), (1,3), (2,6), (3,9)\} \pmod{-39} \\ P_5 &= \{(0,9), (1,6), (2,3), (3,0)\} \pmod{-39} \\ P_6 &= \{(0,0), (1,4), (2,8), (3,12)\} \pmod{-39} \\ P_7 &= \{(0,12), (1,8), (2,4), (3,0)\} \pmod{-39}. \end{aligned}$$

It is well known that there exists an RPBD(3; 39) with 19 parallel classes. Place a copy of this design on each Z_{39} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{39} the parallel class is placed and $j = 1, \dots, 19$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{aligned} &\{(0,0),(1,31),(2,19)\} \pmod{-39} \cup R_{3,1} && \{(0,0),(1,34),(3,25)\} \pmod{-39} \cup R_{2,1} \\ &\{(0,0),(1,5),(2,26)\} \pmod{-39} \cup R_{3,2} && \{(0,0),(1,38),(3,20)\} \pmod{-39} \cup R_{2,2} \\ &\{(0,0),(1,13),(2,22)\} \pmod{-39} \cup R_{3,3} && \{(0,0),(1,19),(3,29)\} \pmod{-39} \cup R_{2,3} \\ &\{(0,0),(1,29),(2,12)\} \pmod{-39} \cup R_{3,4} && \{(0,0),(1,18),(3,23)\} \pmod{-39} \cup R_{2,4} \\ &\{(0,0),(1,9),(2,17)\} \pmod{-39} \cup R_{3,5} && \{(0,0),(1,23),(3,3)\} \pmod{-39} \cup R_{2,5} \\ &\{(0,0),(1,1),(2,20)\} \pmod{-39} \cup R_{3,6} && \{(0,0),(1,6),(3,15)\} \pmod{-39} \cup R_{2,6} \\ &\{(0,0),(1,16),(2,15)\} \pmod{-39} \cup R_{3,7} && \{(0,0),(1,28),(3,16)\} \pmod{-39} \cup R_{2,7} \\ &\{(0,0),(1,14),(2,3)\} \pmod{-39} \cup R_{3,8} && \{(0,0),(1,25),(3,36)\} \pmod{-39} \cup R_{2,8} \\ &\{(0,0),(1,12),(2,18)\} \pmod{-39} \cup R_{3,9} && \{(0,0),(1,10),(3,32)\} \pmod{-39} \cup R_{2,9} \\ &\{(0,0),(1,20),(2,14)\} \pmod{-39} \cup R_{3,10} && \{(0,0),(1,24),(3,1)\} \pmod{-39} \cup R_{2,10} \\ &\{(0,0),(1,17),(2,9)\} \pmod{-39} \cup R_{3,11} && \{(0,0),(1,11),(3,24)\} \pmod{-39} \cup R_{2,11} \\ &\{(0,0),(1,21),(2,7)\} \pmod{-39} \cup R_{3,12} && \{(0,0),(1,26),(3,7)\} \pmod{-39} \cup R_{2,12} \\ &\{(0,0),(1,22),(2,37)\} \pmod{-39} \cup R_{3,13} && \{(0,0),(1,7),(3,35)\} \pmod{-39} \cup R_{2,13} \\ &\{(0,0),(1,15),(2,10)\} \pmod{-39} \cup R_{3,14} && \{(0,0),(1,33),(3,26)\} \pmod{-39} \cup R_{2,14} \\ &\{(0,0),(1,8),(2,25)\} \pmod{-39} \cup R_{3,15} && \{(0,0),(1,27),(3,17)\} \pmod{-39} \cup R_{2,15} \\ &\{(0,0),(1,32),(2,13)\} \pmod{-39} \cup R_{3,16} && \{(0,0),(1,30),(3,5)\} \pmod{-39} \cup R_{2,16} \\ \\ &\{(0,0),(2,36),(3,10)\} \pmod{-39} \cup R_{1,1} && \{(1,0),(2,16),(3,37)\} \pmod{-39} \cup R_{0,1} \\ &\{(0,0),(2,24),(3,18)\} \pmod{-39} \cup R_{1,2} && \{(1,0),(2,5),(3,23)\} \pmod{-39} \cup R_{0,2} \\ &\{(0,0),(2,30),(3,13)\} \pmod{-39} \cup R_{1,3} && \{(1,0),(2,7),(3,38)\} \pmod{-39} \cup R_{0,3} \\ &\{(0,0),(2,32),(3,31)\} \pmod{-39} \cup R_{1,4} && \{(1,0),(2,26),(3,15)\} \pmod{-39} \cup R_{0,4} \\ &\{(0,0),(2,29),(3,34)\} \pmod{-39} \cup R_{1,5} && \{(1,0),(2,1),(3,17)\} \pmod{-39} \cup R_{0,5} \\ &\{(0,0),(2,34),(3,4)\} \pmod{-39} \cup R_{1,6} && \{(1,0),(2,29),(3,24)\} \pmod{-39} \cup R_{0,6} \\ &\{(0,0),(2,1),(3,11)\} \pmod{-39} \cup R_{1,7} && \{(1,0),(2,14),(3,34)\} \pmod{-39} \cup R_{0,7} \\ &\{(0,0),(2,23),(3,8)\} \pmod{-39} \cup R_{1,8} && \{(1,0),(2,18),(3,26)\} \pmod{-39} \cup R_{0,8} \\ &\{(0,0),(2,2),(3,21)\} \pmod{-39} \cup R_{1,9} && \{(1,0),(2,24),(3,12)\} \pmod{-39} \cup R_{0,9} \\ &\{(0,0),(2,38),(3,14)\} \pmod{-39} \cup R_{1,10} && \{(1,0),(2,13),(3,25)\} \pmod{-39} \cup R_{0,10} \\ &\{(0,0),(2,11),(3,37)\} \pmod{-39} \cup R_{1,11} && \{(1,0),(2,30),(3,36)\} \pmod{-39} \cup R_{0,11} \\ &\{(0,0),(2,28),(3,19)\} \pmod{-39} \cup R_{1,12} && \{(1,0),(2,12),(3,2)\} \pmod{-39} \cup R_{0,12} \\ &\{(0,0),(2,16),(3,2)\} \pmod{-39} \cup R_{1,13} && \{(1,0),(2,10),(3,3)\} \pmod{-39} \cup R_{0,13} \\ &\{(0,0),(2,21),(3,22)\} \pmod{-39} \cup R_{1,14} && \{(1,0),(2,32),(3,7)\} \pmod{-39} \cup R_{0,14} \\ &\{(0,0),(2,5),(3,28)\} \pmod{-39} \cup R_{1,15} && \{(1,0),(2,11),(3,18)\} \pmod{-39} \cup R_{0,15} \\ &\{(0,0),(2,27),(3,38)\} \pmod{-39} \cup R_{1,16} && \{(1,0),(2,23),(3,1)\} \pmod{-39} \cup R_{0,16}. \end{aligned}$$

The last three parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,17}$, $\bigcup_{i=0}^3 R_{i,18}$ and $\bigcup_{i=0}^3 R_{i,19}$.

Example A.11 There exists a URD($\{3, 4\}; 156$) with $r_4 = 9$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{39}$. Take the following 9 parallel classes with blocks of size 4:

$$\begin{aligned} P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-39} \\ P_2 &= \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-39} \\ P_3 &= \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-39} \\ P_4 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-39} \\ P_5 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-39} \\ P_6 &= \{(0,0), (1,4), (2,8), (3,13)\} \pmod{-39} \\ P_7 &= \{(0,13), (1,9), (2,5), (3,0)\} \pmod{-39} \\ P_8 &= \{(0,0), (1,8), (2,16), (3,24)\} \pmod{-39} \\ P_9 &= \{(0,24), (1,16), (2,8), (3,0)\} \pmod{-39} \end{aligned}$$

It is well known that there is an RPBD(3; 39) with 19 parallel classes. Place a copy of this design on each Z_{39} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{39} the parallel class is placed and $j = 1, \dots, 19$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{aligned} &\{(0,0),(1,3),(2,25)\} \pmod{-39} \cup R_{3,1} && \{(0,0),(1,16),(3,30)\} \pmod{-39} \cup R_{2,1} \\ &\{(0,0),(1,17),(2,38)\} \pmod{-39} \cup R_{3,2} && \{(0,0),(1,12),(3,20)\} \pmod{-39} \cup R_{2,2} \\ &\{(0,0),(1,7),(2,12)\} \pmod{-39} \cup R_{3,3} && \{(0,0),(1,9),(3,4)\} \pmod{-39} \cup R_{2,3} \\ &\{(0,0),(1,25),(2,5)\} \pmod{-39} \cup R_{3,4} && \{(0,0),(1,34),(3,27)\} \pmod{-39} \cup R_{2,4} \\ &\{(0,0),(1,15),(2,26)\} \pmod{-39} \cup R_{3,5} && \{(0,0),(1,22),(3,1)\} \pmod{-39} \cup R_{2,5} \\ &\{(0,0),(1,21),(2,33)\} \pmod{-39} \cup R_{3,6} && \{(0,0),(1,29),(3,21)\} \pmod{-39} \cup R_{2,6} \\ &\{(0,0),(1,14),(2,9)\} \pmod{-39} \cup R_{3,7} && \{(0,0),(1,24),(3,12)\} \pmod{-39} \cup R_{2,7} \\ &\{(0,0),(1,18),(2,32)\} \pmod{-39} \cup R_{3,8} && \{(0,0),(1,36),(3,17)\} \pmod{-39} \cup R_{2,8} \\ &\{(0,0),(1,20),(2,7)\} \pmod{-39} \cup R_{3,9} && \{(0,0),(1,30),(3,19)\} \pmod{-39} \cup R_{2,9} \\ &\{(0,0),(1,13),(2,6)\} \pmod{-39} \cup R_{3,10} && \{(0,0),(1,27),(3,34)\} \pmod{-39} \cup R_{2,10} \\ &\{(0,0),(1,11),(2,24)\} \pmod{-39} \cup R_{3,11} && \{(0,0),(1,23),(3,9)\} \pmod{-39} \cup R_{2,11} \\ &\{(0,0),(1,26),(2,20)\} \pmod{-39} \cup R_{3,12} && \{(0,0),(1,33),(3,5)\} \pmod{-39} \cup R_{2,12} \\ &\{(0,0),(1,10),(2,34)\} \pmod{-39} \cup R_{3,13} && \{(0,0),(1,5),(3,22)\} \pmod{-39} \cup R_{2,13} \\ &\{(0,0),(1,32),(2,11)\} \pmod{-39} \cup R_{3,14} && \{(0,0),(1,28),(3,29)\} \pmod{-39} \cup R_{2,14} \\ &\{(0,0),(1,6),(2,29)\} \pmod{-39} \cup R_{3,15} && \{(0,0),(1,19),(3,18)\} \pmod{-39} \cup R_{2,15} \\ \\ &\{(0,0),(2,21),(3,11)\} \pmod{-39} \cup R_{1,1} && \{(0,0),(2,36),(3,14)\} \pmod{-39} \cup R_{1,15} \\ &\{(0,0),(2,10),(3,16)\} \pmod{-39} \cup R_{1,2} && \\ &\{(0,0),(2,1),(3,31)\} \pmod{-39} \cup R_{1,3} && \{(1,0),(2,6),(3,3)\} \pmod{-39} \cup R_{0,1} \\ &\{(0,0),(2,17),(3,38)\} \pmod{-39} \cup R_{1,4} && \{(1,0),(2,29),(3,15)\} \pmod{-39} \cup R_{0,2} \\ &\{(0,0),(2,27),(3,10)\} \pmod{-39} \cup R_{1,5} && \{(1,0),(2,15),(3,22)\} \pmod{-39} \cup R_{0,3} \\ &\{(0,0),(2,13),(3,28)\} \pmod{-39} \cup R_{1,6} && \{(1,0),(2,36),(3,21)\} \pmod{-39} \cup R_{0,4} \\ &\{(0,0),(2,28),(3,37)\} \pmod{-39} \cup R_{1,7} && \{(1,0),(2,28),(3,12)\} \pmod{-39} \cup R_{0,5} \\ &\{(0,0),(2,3),(3,23)\} \pmod{-39} \cup R_{1,8} && \{(1,0),(2,7),(3,33)\} \pmod{-39} \cup R_{0,6} \\ &\{(0,0),(2,14),(3,7)\} \pmod{-39} \cup R_{1,9} && \{(1,0),(2,10),(3,26)\} \pmod{-39} \cup R_{0,7} \\ &\{(0,0),(2,30),(3,2)\} \pmod{-39} \cup R_{1,10} && \{(1,0),(2,20),(3,24)\} \pmod{-39} \cup R_{0,8} \\ &\{(0,0),(2,15),(3,25)\} \pmod{-39} \cup R_{1,11} && \{(1,0),(2,3),(3,36)\} \pmod{-39} \cup R_{0,9} \\ &\{(0,0),(2,22),(3,35)\} \pmod{-39} \cup R_{1,12} && \{(1,0),(2,30),(3,10)\} \pmod{-39} \cup R_{0,10} \\ &\{(0,0),(2,19),(3,8)\} \pmod{-39} \cup R_{1,13} && \{(1,0),(2,27),(3,6)\} \pmod{-39} \cup R_{0,11} \\ &\{(0,0),(2,18),(3,32)\} \pmod{-39} \cup R_{1,14} && \{(1,0),(2,16),(3,19)\} \pmod{-39} \cup R_{0,12} \\ &&& \{(1,0),(2,25),(3,13)\} \pmod{-39} \cup R_{0,13} \end{aligned}$$

$$\{(1,0),(2,9),(3,5)\} \pmod{-39} \cup R_{0,14}$$

$$\{(1,0),(2,17),(3,29)\} \pmod{-39} \cap R_{0,15}$$

The last four parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,16}, \bigcup_{i=0}^3 R_{i,17}, \bigcup_{i=0}^3 R_{i,18}$ and $\bigcup_{i=0}^3 R_{i,19}$.

Example A.12 There exists a URD($\{3, 4\}; 204$) with $r_4 = 7$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{51}$. Take the following seven parallel classes with blocks of size 4:

$$P_1 = \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-51}$$

$$P_2 = \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-51}$$

$$P_3 = \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-51}$$

$$P_4 = \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-51}$$

$$P_5 = \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-51}$$

$$P_6 = \{(0,0), (1,4), (2,8), (3,13)\} \pmod{-51}$$

$$P_7 = \{(0,13), (1,9), (2,5), (3,0)\} \pmod{-51}$$

It is well known that there is an RPBD(3; 51) with 25 parallel classes. Place a copy of this design on each Z_{51} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{51} the parallel class is placed and $j = 1, \dots, 25$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{aligned} &\{(0,0),(1,37),(2,45)\} \pmod{-51} \cup R_{3,1} \\ &\{(0,0),(1,3),(2,29)\} \pmod{-51} \cup R_{3,2} \\ &\{(0,0),(1,45),(2,23)\} \pmod{-51} \cup R_{3,3} \\ &\{(0,0),(1,21),(2,38)\} \pmod{-51} \cup R_{3,4} \\ &\{(0,0),(1,26),(2,18)\} \pmod{-51} \cup R_{3,5} \\ &\{(0,0),(1,36),(2,5)\} \pmod{-51} \cup R_{3,6} \\ &\{(0,0),(1,27),(2,3)\} \pmod{-51} \cup R_{3,7} \\ &\{(0,0),(1,43),(2,36)\} \pmod{-51} \cup R_{3,8} \\ &\{(0,0),(1,9),(2,42)\} \pmod{-51} \cup R_{3,9} \\ &\{(0,0),(1,32),(2,50)\} \pmod{-51} \cup R_{3,10} \\ &\{(0,0),(1,29),(2,44)\} \pmod{-51} \cup R_{3,11} \\ &\{(0,0),(1,40),(2,30)\} \pmod{-51} \cup R_{3,12} \\ &\{(0,0),(1,18),(2,48)\} \pmod{-51} \cup R_{3,13} \\ &\{(0,0),(1,34),(2,39)\} \pmod{-51} \cup R_{3,14} \\ &\{(0,0),(1,23),(2,34)\} \pmod{-51} \cup R_{3,15} \\ &\{(0,0),(1,10),(2,35)\} \pmod{-51} \cup R_{3,16} \\ &\{(0,0),(1,24),(2,33)\} \pmod{-51} \cup R_{3,17} \\ &\{(0,0),(1,35),(2,32)\} \pmod{-51} \cup R_{3,18} \\ &\{(0,0),(1,11),(2,46)\} \pmod{-51} \cup R_{3,19} \\ &\{(0,0),(1,38),(2,24)\} \pmod{-51} \cup R_{3,20} \\ &\{(0,0),(1,48),(2,37)\} \pmod{-51} \cup R_{3,21} \\ &\{(0,0),(1,15),(2,31)\} \pmod{-51} \cup R_{3,22} \end{aligned}$$

$$\begin{aligned} &\{(0,0),(2,22),(3,33)\} \pmod{-51} \cup R_{1,1} \\ &\{(0,0),(2,6),(3,14)\} \pmod{-51} \cup R_{1,2} \\ &\{(0,0),(2,9),(3,44)\} \pmod{-51} \cup R_{1,3} \\ &\{(0,0),(2,26),(3,30)\} \pmod{-51} \cup R_{1,4} \\ &\{(0,0),(2,15),(3,18)\} \pmod{-51} \cup R_{1,5} \\ &\{(0,0),(2,21),(3,49)\} \pmod{-51} \cup R_{1,6} \end{aligned}$$

$$\begin{aligned} &\{(0,0),(1,25),(3,35)\} \pmod{-51} \cup R_{2,1} \\ &\{(0,0),(1,42),(3,36)\} \pmod{-51} \cup R_{2,2} \\ &\{(0,0),(1,7),(3,42)\} \pmod{-51} \cup R_{2,3} \\ &\{(0,0),(1,33),(3,11)\} \pmod{-51} \cup R_{2,4} \\ &\{(0,0),(1,5),(3,32)\} \pmod{-51} \cup R_{2,5} \\ &\{(0,0),(1,8),(3,19)\} \pmod{-51} \cup R_{2,6} \\ &\{(0,0),(1,19),(3,34)\} \pmod{-51} \cup R_{2,7} \\ &\{(0,0),(1,12),(3,5)\} \pmod{-51} \cup R_{2,8} \\ &\{(0,0),(1,46),(3,43)\} \pmod{-51} \cup R_{2,9} \\ &\{(0,0),(1,16),(3,50)\} \pmod{-51} \cup R_{2,10} \\ &\{(0,0),(1,13),(3,2)\} \pmod{-51} \cup R_{2,11} \\ &\{(0,0),(1,30),(3,46)\} \pmod{-51} \cup R_{2,12} \\ &\{(0,0),(1,28),(3,29)\} \pmod{-51} \cup R_{2,13} \\ &\{(0,0),(1,31),(3,37)\} \pmod{-51} \cup R_{2,14} \\ &\{(0,0),(1,39),(3,16)\} \pmod{-51} \cup R_{2,15} \\ &\{(0,0),(1,44),(3,26)\} \pmod{-51} \cup R_{2,16} \\ &\{(0,0),(1,17),(3,41)\} \pmod{-51} \cup R_{2,17} \\ &\{(0,0),(1,6),(3,23)\} \pmod{-51} \cup R_{2,18} \\ &\{(0,0),(1,22),(3,12)\} \pmod{-51} \cup R_{2,19} \\ &\{(0,0),(1,41),(3,40)\} \pmod{-51} \cup R_{2,20} \\ &\{(0,0),(1,14),(3,21)\} \pmod{-51} \cup R_{2,21} \\ &\{(0,0),(1,20),(3,28)\} \pmod{-51} \cup R_{2,22} \end{aligned}$$

$$\begin{aligned} &\{(0,0),(2,20),(3,10)\} \pmod{-51} \cup R_{1,7} \\ &\{(0,0),(2,27),(3,15)\} \pmod{-51} \cup R_{1,8} \\ &\{(0,0),(2,10),(3,31)\} \pmod{-51} \cup R_{1,9} \\ &\{(0,0),(2,25),(3,47)\} \pmod{-51} \cup R_{1,10} \\ &\{(0,0),(2,41),(3,24)\} \pmod{-51} \cup R_{1,11} \\ &\{(0,0),(2,16),(3,22)\} \pmod{-51} \cup R_{1,12} \end{aligned}$$

$$\begin{aligned}
& \{(0,0),(2,11),(3,20)\} \pmod{-51} \cup R_{1,13} \\
& \{(0,0),(2,40),(3,8)\} \pmod{-51} \cup R_{1,14} \\
& \{(0,0),(2,7),(3,17)\} \pmod{-51} \cup R_{1,15} \\
& \{(0,0),(2,19),(3,39)\} \pmod{-51} \cup R_{1,16} \\
& \{(0,0),(2,28),(3,4)\} \pmod{-51} \cup R_{1,17} \\
& \{(0,0),(2,13),(3,7)\} \pmod{-51} \cup R_{1,18} \\
& \{(0,0),(2,12),(3,25)\} \pmod{-51} \cup R_{1,19} \\
& \{(0,0),(2,1),(3,27)\} \pmod{-51} \cup R_{1,20} \\
& \{(0,0),(2,14),(3,1)\} \pmod{-51} \cup R_{1,21} \\
& \{(0,0),(2,17),(3,9)\} \pmod{-51} \cup R_{1,22} \\
\\
& \{(1,0),(2,7),(3,23)\} \pmod{-51} \cup R_{0,1} \\
& \{(1,0),(2,14),(3,37)\} \pmod{-51} \cup R_{0,2} \\
& \{(1,0),(2,36),(3,32)\} \pmod{-51} \cup R_{0,3} \\
& \{(1,0),(2,31),(3,43)\} \pmod{-51} \cup R_{0,4} \\
& \{(1,0),(2,6),(3,13)\} \pmod{-51} \cup R_{0,5} \\
& \{(1,0),(2,3),(3,18)\} \pmod{-51} \cup R_{0,6}
\end{aligned}$$

$$\begin{aligned}
& \{(1,0),(2,22),(3,46)\} \pmod{-51} \cup R_{0,7} \\
& \{(1,0),(2,24),(3,3)\} \pmod{-51} \cup R_{0,8} \\
& \{(1,0),(2,23),(3,12)\} \pmod{-51} \cup R_{0,9} \\
& \{(1,0),(2,38),(3,19)\} \pmod{-51} \cup R_{0,10} \\
& \{(1,0),(2,13),(3,31)\} \pmod{-51} \cup R_{0,11} \\
& \{(1,0),(2,10),(3,39)\} \pmod{-51} \cup R_{0,12} \\
& \{(1,0),(2,32),(3,14)\} \pmod{-51} \cup R_{0,13} \\
& \{(1,0),(2,39),(3,36)\} \pmod{-51} \cup R_{0,14} \\
& \{(1,0),(2,34),(3,25)\} \pmod{-51} \cup R_{0,15} \\
& \{(1,0),(2,28),(3,21)\} \pmod{-51} \cup R_{0,16} \\
& \{(1,0),(2,46),(3,20)\} \pmod{-51} \cup R_{0,17} \\
& \{(1,0),(2,45),(3,30)\} \pmod{-51} \cup R_{0,18} \\
& \{(1,0),(2,19),(3,5)\} \pmod{-51} \cup R_{0,19} \\
& \{(1,0),(2,42),(3,22)\} \pmod{-51} \cup R_{0,20} \\
& \{(1,0),(2,21),(3,38)\} \pmod{-51} \cup R_{0,21} \\
& \{(1,0),(2,12),(3,26)\} \pmod{-51} \cup R_{0,22}
\end{aligned}$$

The last three parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,23}$, $\bigcup_{i=0}^3 R_{i,24}$ and $\bigcup_{i=0}^3 R_{i,25}$.

Example A.13 There exists a URD($\{3, 4\}; 204$) with $r_4 = 9$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{51}$. Take the following 9 parallel classes with blocks of size 4:

$$\begin{aligned}
P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-51} \\
P_2 &= \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-51} \\
P_3 &= \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-51} \\
P_4 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-51} \\
P_5 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-51} \\
P_6 &= \{(0,0), (1,4), (2,8), (3,13)\} \pmod{-51} \\
P_7 &= \{(0,13), (1,9), (2,5), (3,0)\} \pmod{-51} \\
P_8 &= \{(0,0), (1,8), (2,16), (3,24)\} \pmod{-51} \\
P_9 &= \{(0,24), (1,16), (2,8), (3,0)\} \pmod{-51}
\end{aligned}$$

It is well known that there is an RPBD(3; 51) with 25 parallel classes. Place a copy of this design on each Z_{51} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{51} the parallel class is placed and $j = 1, \dots, 25$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{aligned} & \{(0,0),(1,41),(2,10)\} \pmod{-51} \cup R_{3,1} \\ & \{(0,0),(1,34),(2,1)\} \pmod{-51} \cup R_{3,2} \\ & \{(0,0),(1,9),(2,32)\} \pmod{-51} \cup R_{3,3} \\ & \{(0,0),(1,33),(2,11)\} \pmod{-51} \cup R_{3,4} \\ & \{(0,0),(1,39),(2,22)\} \pmod{-51} \cup R_{3,5} \\ & \{(0,0),(1,27),(2,48)\} \pmod{-51} \cup R_{3,6} \\ & \{(0,0),(1,15),(2,3)\} \pmod{-51} \cup R_{3,7} \\ & \{(0,0),(1,38),(2,15)\} \pmod{-51} \cup R_{3,8} \\ & \{(0,0),(1,10),(2,40)\} \pmod{-51} \cup R_{3,9} \\ & \{(0,0),(1,35),(2,21)\} \pmod{-51} \cup R_{3,10} \\ & \{(0,0),(1,24),(2,5)\} \pmod{-51} \cup R_{3,11} \\ & \{(0,0),(1,6),(2,37)\} \pmod{-51} \cup R_{3,12} \\ & \{(0,0),(1,14),(2,26)\} \pmod{-51} \cup R_{3,13} \\ & \{(0,0),(1,13),(2,27)\} \pmod{-51} \cup R_{3,14} \\ & \{(0,0),(1,23),(2,34)\} \pmod{-51} \cup R_{3,15} \\ & \{(0,0),(1,3),(2,19)\} \pmod{-51} \cup R_{3,16} \\ & \{(0,0),(1,36),(2,23)\} \pmod{-51} \cup R_{3,17} \\ & \{(0,0),(1,11),(2,14)\} \pmod{-51} \cup R_{3,18} \\ & \{(0,0),(1,40),(2,33)\} \pmod{-51} \cup R_{3,19} \\ & \{(0,0),(1,45),(2,42)\} \pmod{-51} \cup R_{3,20} \\ & \{(0,0),(1,12),(2,17)\} \pmod{-51} \cup R_{3,21} \end{aligned}$$

$$\begin{aligned} & \{(0,0),(2,18),(3,50)\} \pmod{-51} \cup R_{1,1} \\ & \{(0,0),(2,38),(3,21)\} \pmod{-51} \cup R_{1,2} \\ & \{(0,0),(2,6),(3,36)\} \pmod{-51} \cup R_{1,3} \\ & \{(0,0),(2,24),(3,44)\} \pmod{-51} \cup R_{1,4} \\ & \{(0,0),(2,7),(3,31)\} \pmod{-51} \cup R_{1,5} \\ & \{(0,0),(2,41),(3,34)\} \pmod{-51} \cup R_{1,6} \\ & \{(0,0),(2,28),(3,47)\} \pmod{-51} \cup R_{1,7} \\ & \{(0,0),(2,31),(3,11)\} \pmod{-51} \cup R_{1,8} \\ & \{(0,0),(2,20),(3,23)\} \pmod{-51} \cup R_{1,9} \\ & \{(0,0),(2,30),(3,26)\} \pmod{-51} \cup R_{1,10} \\ & \{(0,0),(2,39),(3,29)\} \pmod{-51} \cup R_{1,11} \\ & \{(0,0),(2,50),(3,28)\} \pmod{-51} \cup R_{1,12} \\ & \{(0,0),(2,36),(3,49)\} \pmod{-51} \cup R_{1,13} \\ & \{(0,0),(2,29),(3,4)\} \pmod{-51} \cup R_{1,14} \\ & \{(0,0),(2,45),(3,8)\} \pmod{-51} \cup R_{1,15} \\ & \{(0,0),(2,44),(3,9)\} \pmod{-51} \cup R_{1,16} \\ & \{(0,0),(2,9),(3,18)\} \pmod{-51} \cup R_{1,17} \\ & \{(0,0),(2,46),(3,20)\} \pmod{-51} \cup R_{1,18} \\ & \{(0,0),(2,12),(3,39)\} \pmod{-51} \cup R_{1,19} \\ & \{(0,0),(2,13),(3,46)\} \pmod{-51} \cup R_{1,20} \\ & \{(0,0),(2,25),(3,16)\} \pmod{-51} \cup R_{1,21} \end{aligned}$$

$$\begin{aligned} & \{(0,0),(1,30),(3,17)\} \pmod{-51} \cup R_{2,1} \\ & \{(0,0),(1,22),(3,15)\} \pmod{-51} \cup R_{2,2} \\ & \{(0,0),(1,46),(3,22)\} \pmod{-51} \cup R_{2,3} \\ & \{(0,0),(1,42),(3,41)\} \pmod{-51} \cup R_{2,4} \\ & \{(0,0),(1,21),(3,2)\} \pmod{-51} \cup R_{2,5} \\ & \{(0,0),(1,25),(3,32)\} \pmod{-51} \cup R_{2,6} \\ & \{(0,0),(1,29),(3,37)\} \pmod{-51} \cup R_{2,7} \\ & \{(0,0),(1,26),(3,12)\} \pmod{-51} \cup R_{2,8} \\ & \{(0,0),(1,44),(3,10)\} \pmod{-51} \cup R_{2,9} \\ & \{(0,0),(1,7),(3,35)\} \pmod{-51} \cup R_{2,10} \\ & \{(0,0),(1,37),(3,1)\} \pmod{-51} \cup R_{2,11} \\ & \{(0,0),(1,32),(3,5)\} \pmod{-51} \cup R_{2,12} \\ & \{(0,0),(1,5),(3,25)\} \pmod{-51} \cup R_{2,13} \\ & \{(0,0),(1,17),(3,43)\} \pmod{-51} \cup R_{2,14} \\ & \{(0,0),(1,18),(3,19)\} \pmod{-51} \cup R_{2,15} \\ & \{(0,0),(1,20),(3,14)\} \pmod{-51} \cup R_{2,16} \\ & \{(0,0),(1,28),(3,33)\} \pmod{-51} \cup R_{2,17} \\ & \{(0,0),(1,16),(3,30)\} \pmod{-51} \cup R_{2,18} \\ & \{(0,0),(1,48),(3,40)\} \pmod{-51} \cup R_{2,19} \\ & \{(0,0),(1,31),(3,42)\} \pmod{-51} \cup R_{2,20} \\ & \{(0,0),(1,19),(3,7)\} \pmod{-51} \cup R_{2,21} \end{aligned}$$

$$\begin{aligned} & \{(1,0),(2,19),(3,41)\} \pmod{-51} \cup R_{0,1} \\ & \{(1,0),(2,46),(3,34)\} \pmod{-51} \cup R_{0,2} \\ & \{(1,0),(2,42),(3,46)\} \pmod{-51} \cup R_{0,3} \\ & \{(1,0),(2,26),(3,23)\} \pmod{-51} \cup R_{0,4} \\ & \{(1,0),(2,41),(3,48)\} \pmod{-51} \cup R_{0,5} \\ & \{(1,0),(2,40),(3,12)\} \pmod{-51} \cup R_{0,6} \\ & \{(1,0),(2,24),(3,36)\} \pmod{-51} \cup R_{0,7} \\ & \{(1,0),(2,15),(3,25)\} \pmod{-51} \cup R_{0,8} \\ & \{(1,0),(2,7),(3,18)\} \pmod{-51} \cup R_{0,9} \\ & \{(1,0),(2,27),(3,33)\} \pmod{-51} \cup R_{0,10} \\ & \{(1,0),(2,13),(3,30)\} \pmod{-51} \cup R_{0,11} \\ & \{(1,0),(2,36),(3,13)\} \pmod{-51} \cup R_{0,12} \\ & \{(1,0),(2,17),(3,6)\} \pmod{-51} \cup R_{0,13} \\ & \{(1,0),(2,22),(3,40)\} \pmod{-51} \cup R_{0,14} \\ & \{(1,0),(2,45),(3,29)\} \pmod{-51} \cup R_{0,15} \\ & \{(1,0),(2,25),(3,10)\} \pmod{-51} \cup R_{0,16} \\ & \{(1,0),(2,33),(3,19)\} \pmod{-51} \cup R_{0,17} \\ & \{(1,0),(2,6),(3,21)\} \pmod{-51} \cup R_{0,18} \\ & \{(1,0),(2,35),(3,22)\} \pmod{-51} \cup R_{0,19} \\ & \{(1,0),(2,9),(3,3)\} \pmod{-51} \cup R_{0,20} \\ & \{(1,0),(2,10),(3,31)\} \pmod{-51} \cup R_{0,21} \end{aligned}$$

The last 4 parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,22}, \bigcup_{i=0}^3 R_{i,23}, \bigcup_{i=0}^3 R_{i,24}$ and $\bigcup_{i=0}^3 R_{i,25}$.

Example A.14 There exists a URD($\{3, 4\}; 228$) with $r_4 = 9$.

Proof Let Z_λ be the group of residues modulo λ . The design is constructed on $X = Z_4 \times Z_{57}$. Take the following 9 parallel classes with blocks of size 4:

$$\begin{aligned}
P_1 &= \{(0,0), (1,0), (2,0), (3,0)\} \pmod{-57} \\
P_2 &= \{(0,0), (1,1), (2,2), (3,3)\} \pmod{-57} \\
P_3 &= \{(0,3), (1,2), (2,1), (3,0)\} \pmod{-57} \\
P_4 &= \{(0,0), (1,2), (2,4), (3,6)\} \pmod{-57} \\
P_5 &= \{(0,6), (1,4), (2,2), (3,0)\} \pmod{-57} \\
P_6 &= \{(0,0), (1,4), (2,8), (3,13)\} \pmod{-57} \\
P_7 &= \{(0,13), (1,9), (2,5), (3,0)\} \pmod{-57} \\
P_8 &= \{(0,0), (1,6), (2,12), (3,18)\} \pmod{-57} \\
P_9 &= \{(0,18), (1,12), (2,6), (3,0)\} \pmod{-57}
\end{aligned}$$

It is well known that there is an RPBD(3; 57) with 28 parallel classes. Place a copy of this design on each Z_{57} set. Denote the resolution classes by $R_{i,j}$ where $i \in Z_4$ denotes on which copy of Z_{57} the parallel class is placed and $j = 1, \dots, 28$ are the resolution classes. The parallel classes of the triples are formed as follows:

$$\begin{aligned}
&\{(0,0),(1,5),(2,23)\} \pmod{-57} \cup R_{3,1} && \{(0,0),(1,40),(3,9)\} \pmod{-57} \cup R_{2,1} \\
&\{(0,0),(1,44),(2,1)\} \pmod{-57} \cup R_{3,2} && \{(0,0),(1,17),(3,20)\} \pmod{-57} \cup R_{2,2} \\
&\{(0,0),(1,37),(2,47)\} \pmod{-57} \cup R_{3,3} && \{(0,0),(1,48),(3,21)\} \pmod{-57} \cup R_{2,3} \\
&\{(0,0),(1,41),(2,33)\} \pmod{-57} \cup R_{3,4} && \{(0,0),(1,8),(3,36)\} \pmod{-57} \cup R_{2,4} \\
&\{(0,0),(1,14),(2,3)\} \pmod{-57} \cup R_{3,5} && \{(0,0),(1,26),(3,23)\} \pmod{-57} \cup R_{2,5} \\
&\{(0,0),(1,12),(2,31)\} \pmod{-57} \cup R_{3,6} && \{(0,0),(1,28),(3,43)\} \pmod{-57} \cup R_{2,6} \\
&\{(0,0),(1,29),(2,44)\} \pmod{-57} \cup R_{3,7} && \{(0,0),(1,52),(3,45)\} \pmod{-57} \cup R_{2,7} \\
&\{(0,0),(1,11),(2,38)\} \pmod{-57} \cup R_{3,8} && \{(0,0),(1,47),(3,22)\} \pmod{-57} \cup R_{2,8} \\
&\{(0,0),(1,25),(2,37)\} \pmod{-57} \cup R_{3,9} && \{(0,0),(1,22),(3,49)\} \pmod{-57} \cup R_{2,9} \\
&\{(0,0),(1,32),(2,39)\} \pmod{-57} \cup R_{3,10} && \{(0,0),(1,16),(3,10)\} \pmod{-57} \cup R_{2,10} \\
&\{(0,0),(1,19),(2,40)\} \pmod{-57} \cup R_{3,11} && \{(0,0),(1,34),(3,42)\} \pmod{-57} \cup R_{2,11} \\
&\{(0,0),(1,21),(2,52)\} \pmod{-57} \cup R_{3,12} && \{(0,0),(1,49),(3,14)\} \pmod{-57} \cup R_{2,12} \\
&\{(0,0),(1,42),(2,18)\} \pmod{-57} \cup R_{3,13} && \{(0,0),(1,36),(3,50)\} \pmod{-57} \cup R_{2,13} \\
&\{(0,0),(1,38),(2,10)\} \pmod{-57} \cup R_{3,14} && \{(0,0),(1,18),(3,29)\} \pmod{-57} \cup R_{2,14} \\
&\{(0,0),(1,24),(2,14)\} \pmod{-57} \cup R_{3,15} && \{(0,0),(1,33),(3,7)\} \pmod{-57} \cup R_{2,15} \\
&\{(0,0),(1,9),(2,48)\} \pmod{-57} \cup R_{3,16} && \{(0,0),(1,54),(3,2)\} \pmod{-57} \cup R_{2,16} \\
&\{(0,0),(1,27),(2,22)\} \pmod{-57} \cup R_{3,17} && \{(0,0),(1,3),(3,16)\} \pmod{-57} \cup R_{2,17} \\
&\{(0,0),(1,13),(2,43)\} \pmod{-57} \cup R_{3,18} && \{(0,0),(1,35),(3,53)\} \pmod{-57} \cup R_{2,18} \\
&\{(0,0),(1,46),(2,11)\} \pmod{-57} \cup R_{3,19} && \{(0,0),(1,15),(3,32)\} \pmod{-57} \cup R_{2,19} \\
&\{(0,0),(1,45),(2,20)\} \pmod{-57} \cup R_{3,20} && \{(0,0),(1,39),(3,1)\} \pmod{-57} \cup R_{2,20} \\
&\{(0,0),(1,23),(2,46)\} \pmod{-57} \cup R_{3,21} && \{(0,0),(1,7),(3,56)\} \pmod{-57} \cup R_{2,21} \\
&\{(0,0),(1,20),(2,5)\} \pmod{-57} \cup R_{3,22} && \{(0,0),(1,31),(3,37)\} \pmod{-57} \cup R_{2,22} \\
&\{(0,0),(1,50),(2,21)\} \pmod{-57} \cup R_{3,23} && \{(0,0),(1,30),(3,31)\} \pmod{-57} \cup R_{2,23} \\
&\{(0,0),(1,10),(2,36)\} \pmod{-57} \cup R_{3,24} && \{(0,0),(1,43),(3,24)\} \pmod{-57} \cup R_{2,24}
\end{aligned}$$

$\{(0,0),(2,32),(3,55)\} \pmod{-57} \cup R_{1,1}$
 $\{(0,0),(2,51),(3,48)\} \pmod{-57} \cup R_{1,2}$
 $\{(0,0),(2,27),(3,19)\} \pmod{-57} \cup R_{1,3}$
 $\{(0,0),(2,25),(3,4)\} \pmod{-57} \cup R_{1,14}$
 $\{(0,0),(2,41),(3,52)\} \pmod{-57} \cup R_{1,5}$
 $\{(0,0),(2,19),(3,34)\} \pmod{-57} \cup R_{1,6}$
 $\{(0,0),(2,28),(3,47)\} \pmod{-57} \cup R_{1,7}$
 $\{(0,0),(2,17),(3,8)\} \pmod{-57} \cup R_{1,8}$
 $\{(0,0),(2,34),(3,12)\} \pmod{-57} \cup R_{1,9}$
 $\{(0,0),(2,30),(3,17)\} \pmod{-57} \cup R_{1,10}$
 $\{(0,0),(2,42),(3,25)\} \pmod{-57} \cup R_{1,11}$
 $\{(0,0),(2,54),(3,27)\} \pmod{-57} \cup R_{1,12}$
 $\{(0,0),(2,29),(3,38)\} \pmod{-57} \cup R_{1,13}$
 $\{(0,0),(2,6),(3,35)\} \pmod{-57} \cup R_{1,14}$
 $\{(0,0),(2,15),(3,33)\} \pmod{-57} \cup R_{1,15}$
 $\{(0,0),(2,35),(3,15)\} \pmod{-57} \cup R_{1,16}$
 $\{(0,0),(2,56),(3,40)\} \pmod{-57} \cup R_{1,17}$
 $\{(0,0),(2,7),(3,28)\} \pmod{-57} \cup R_{1,18}$
 $\{(0,0),(2,13),(3,41)\} \pmod{-57} \cup R_{1,19}$
 $\{(0,0),(2,9),(3,5)\} \pmod{-57} \cup R_{1,20}$
 $\{(0,0),(2,50),(3,26)\} \pmod{-57} \cup R_{1,21}$
 $\{(0,0),(2,24),(3,46)\} \pmod{-57} \cup R_{1,22}$
 $\{(0,0),(2,16),(3,30)\} \pmod{-57} \cup R_{1,23}$
 $\{(0,0),(2,26),(3,11)\} \pmod{-57} \cup R_{1,24}$

$\{(1,0),(2,16),(3,36)\} \pmod{-57} \cup R_{0,1}$
 $\{(1,0),(2,9),(3,35)\} \pmod{-57} \cup R_{0,2}$
 $\{(1,0),(2,48),(3,16)\} \pmod{-57} \cup R_{0,3}$
 $\{(1,0),(2,34),(3,24)\} \pmod{-57} \cup R_{0,4}$
 $\{(1,0),(2,38),(3,20)\} \pmod{-57} \cup R_{0,5}$
 $\{(1,0),(2,20),(3,23)\} \pmod{-57} \cup R_{0,6}$
 $\{(1,0),(2,50),(3,25)\} \pmod{-57} \cup R_{0,7}$
 $\{(1,0),(2,13),(3,56)\} \pmod{-57} \cup R_{0,8}$
 $\{(1,0),(2,54),(3,47)\} \pmod{-57} \cup R_{0,9}$
 $\{(1,0),(2,25),(3,37)\} \pmod{-57} \cup R_{0,10}$
 $\{(1,0),(2,24),(3,40)\} \pmod{-57} \cup R_{0,11}$
 $\{(1,0),(2,43),(3,10)\} \pmod{-57} \cup R_{0,12}$
 $\{(1,0),(2,41),(3,29)\} \pmod{-57} \cup R_{0,13}$
 $\{(1,0),(2,8),(3,21)\} \pmod{-57} \cup R_{0,14}$
 $\{(1,0),(2,45),(3,52)\} \pmod{-57} \cup R_{0,15}$
 $\{(1,0),(2,40),(3,44)\} \pmod{-57} \cup R_{0,16}$
 $\{(1,0),(2,17),(3,34)\} \pmod{-57} \cup R_{0,17}$
 $\{(1,0),(2,5),(3,39)\} \pmod{-57} \cup R_{0,18}$
 $\{(1,0),(2,44),(3,33)\} \pmod{-57} \cup R_{0,19}$
 $\{(1,0),(2,3),(3,41)\} \pmod{-57} \cup R_{0,20}$
 $\{(1,0),(2,35),(3,43)\} \pmod{-57} \cup R_{0,21}$
 $\{(1,0),(2,36),(3,46)\} \pmod{-57} \cup R_{0,22}$
 $\{(1,0),(2,37),(3,7)\} \pmod{-57} \cup R_{0,23}$
 $\{(1,0),(2,11),(3,42)\} \pmod{-57} \cup R_{0,24}$

The last 4 parallel classes of triples are given by $\bigcup_{i=0}^3 R_{i,25}$, $\bigcup_{i=0}^3 R_{i,26}$, $\bigcup_{i=0}^3 R_{i,27}$ and $\bigcup_{i=0}^3 R_{i,28}$.

Example A.15 A uniform $\{2, 4\}$ -LRGDD₂ of type 2^6 with $r_2 = 5$ and $r_4 = 5$,

$G = \{\{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\}, \{11, 12\}\}$; each row forms a uniform parallel class:

(5 11; 0), (4 10; 0), (3 9; 0), (2 6; 0), (7 12; 0), (1 8; 1),
 (4 5; 1), (8 12; 0), (6 10; 1), (1 11; 0), (7 9; 1), (2 3; 0),
 (6 8; 1), (9 11; 0), (2 4; 1), (7 10; 1), (1 5; 0), (3 12; 1),
 (3 8; 0), (10 11; 0), (1 5; 1), (4 12; 0), (2 9; 0), (6 7; 1),
 (5 10; 1), (1 11; 1), (4 6; 1), (2 3; 1), (7 12; 1), (8 9; 1),
 (1 6 9 12; 1 1 1 0 0 0), (2 4 7 11; 0 0 0 0 0 0), (3 5 8 10; 1 1 1 0 0 0),
 (2 5 8 12; 1 0 1 1 0 1), (3 6 9 11; 0 1 0 1 0 1), (1 4 7 10; 0 1 1 1 1 0),
 (2 8 10 11; 1 0 1 1 0 1), (4 5 9 12; 0 0 1 0 1 1), (1 3 6 7; 1 0 0 1 1 0),
 (2 5 7 9; 0 1 1 1 1 0), (4 6 8 11; 0 0 1 0 1 1), (1 3 10 12; 0 0 0 0 0 0),
 (1 4 8 9; 1 0 0 1 1 0), (2 6 10 12; 1 1 0 0 1 1), (3 5 7 11; 0 0 1 0 1 1).

With Theorem 2.1 we obtain a URD($\{2, 4\}; 24$) with $r_2 = 5$.

Example A.16 A uniform $\{3, 5\}$ -LRGDD₇ of type 3^5 with $r_3 = 38$ and $r_5 = 2$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}, \{13, 14, 15\}\}$; each row forms a uniform parallel class:

(1 11 13; 4 5 1), (3 6 9; 1 6 5), (4 7 12; 0 3 3), (2 10 14; 6 2 3), (5 8 15; 4 1 4),
 (2 9 15; 6 0 1), (1 7 12; 4 4 0), (5 8 14; 6 6 0), (3 6 11; 6 0 1), (4 10 13; 4 0 3),
 (3 6 11; 2 4 2), (9 10 15; 3 6 3), (2 8 12; 1 3 2), (1 4 14; 2 5 3), (5 7 13; 1 1 0),
 (5 7 10; 5 3 5), (3 6 13; 3 2 6), (4 8 14; 2 4 2), (2 9 11; 5 3 5), (1 12 15; 5 4 6),
 (2 7 15; 0 3 3), (1 5 11; 0 2 2), (4 12 13; 1 1 0), (6 9 10; 2 3 1), (3 8 14; 2 6 4),
 (3 7 15; 3 3 0), (2 5 10; 5 2 4), (9 11 13; 4 3 6), (6 8 12; 2 6 4), (1 4 14; 6 4 5),
 (3 6 12; 0 1 1), (9 10 13; 5 2 4), (2 5 11; 3 2 6), (4 8 15; 5 5 0), (1 7 14; 0 0 0),
 (2 11 13; 0 3 3), (8 12 14; 6 5 6), (3 4 9; 4 0 3), (1 5 7; 6 1 2), (6 10 15; 2 0 5),
 (3 8 15; 3 4 1), (4 11 14; 3 0 4), (1 6 10; 5 4 6), (2 7 12; 4 5 1), (5 9 13; 2 0 5),
 (1 8 13; 4 4 0), (3 4 7; 2 4 2), (2 6 12; 5 0 2), (9 10 14; 2 1 6), (5 11 15; 3 2 6),
 (2 4 7; 3 1 5), (5 8 10; 3 6 3), (1 6 13; 3 1 5), (3 12 14; 5 2 4), (9 11 15; 0 3 3),
 (3 6 15; 4 0 3), (1 7 14; 2 1 6), (2 8 10; 0 5 5), (4 11 13; 1 6 5), (5 9 12; 1 3 2),
 (3 6 12; 5 3 5), (5 9 13; 6 6 0), (2 7 14; 3 6 3), (4 10 15; 5 0 2), (1 8 11; 2 6 4),
 (2 9 13; 4 1 4), (3 5 12; 3 4 1), (7 11 15; 1 5 4), (1 4 8; 5 6 1), (6 10 14; 0 4 4),
 (4 7 10; 1 0 6), (3 5 8; 0 0 0), (2 12 15; 1 1 0), (9 11 13; 1 1 0), (1 6 14; 0 2 2),
 (1 6 15; 1 3 2), (2 4 9; 5 0 2), (7 10 13; 4 4 0), (5 8 12; 2 5 3), (3 11 14; 6 1 2),
 (4 9 12; 5 4 6), (7 11 14; 6 5 6), (2 8 15; 2 4 2), (3 5 13; 2 0 5), (1 6 10; 4 2 5),
 (6 9 14; 4 0 3), (1 4 11; 1 0 6), (2 5 15; 0 6 6), (8 12 13; 5 3 5), (3 7 10; 1 2 1),
 (3 12 14; 2 5 3), (6 8 15; 5 1 3), (1 9 11; 6 1 2), (2 5 13; 6 2 3), (4 7 10; 6 6 0),
 (9 12 14; 4 4 0), (2 6 13; 1 4 3), (3 8 10; 1 5 4), (1 5 11; 3 3 0), (4 7 15; 4 1 4),
 (7 12 13; 2 3 1), (2 10 14; 4 4 0), (1 4 9; 0 0 0), (6 8 11; 6 4 5), (3 5 15; 1 5 4),
 (3 7 14; 5 0 2), (4 9 11; 4 0 3), (5 12 15; 6 3 4), (1 10 13; 5 6 1), (2 6 8; 6 3 4),
 (2 8 11; 4 6 2), (6 9 14; 3 1 5), (1 12 15; 1 2 1), (5 7 10; 3 5 2), (3 4 13; 6 1 2),
 (3 7 12; 2 0 5), (2 4 9; 1 2 1), (1 8 15; 3 1 5), (6 10 13; 4 2 5), (5 11 14; 1 2 1),
 (3 9 13; 5 4 6), (4 8 10; 3 3 0), (2 5 15; 4 2 5), (1 12 14; 2 3 1), (6 7 11; 2 0 5),
 (5 12 14; 0 5 5), (2 7 11; 2 5 3), (6 8 13; 0 1 1), (1 10 15; 1 0 6), (3 4 9; 5 4 6),
 (6 7 14; 5 6 1), (3 5 9; 5 1 3), (4 12 15; 2 4 2), (1 8 10; 5 6 1), (2 11 13; 4 6 2),
 (1 5 9; 1 1 0), (3 10 15; 1 1 0), (4 8 14; 0 1 1), (6 12 13; 4 0 3), (2 7 11; 6 1 2),
 (3 10 13; 0 6 6), (6 7 11; 1 5 4), (4 8 15; 4 3 6), (1 9 12; 4 0 3), (2 5 14; 2 3 1),
 (2 4 14; 2 1 6), (6 7 13; 6 4 5), (3 11 15; 1 6 5), (5 8 10; 1 0 6), (1 9 12; 5 6 1),
 (5 11 14; 5 3 5), (1 6 8; 6 0 1), (3 7 13; 6 5 6), (2 4 12; 6 4 5), (9 10 15; 6 0 1),
 (1 8 13; 1 3 2), (3 4 11; 0 5 5), (5 12 15; 4 0 3), (2 6 9; 3 3 0), (7 10 14; 3 4 1),
 (1 10 13; 0 2 2), (3 8 12; 5 6 1), (4 11 15; 2 2 0), (2 5 7; 1 5 4), (6 9 14; 6 5 6),
 (6 7 12; 4 3 6), (1 5 9; 5 2 4), (4 11 15; 4 6 2), (2 10 14; 3 5 2), (3 8 13; 6 3 4),
 (1 5 10; 2 3 1), (3 7 15; 0 2 2), (2 4 13; 4 0 3), (9 12 14; 5 0 2), (6 8 11; 3 6 3),
 (8 11 14; 0 3 3), (5 7 13; 0 2 2), (2 6 12; 2 2 0), (3 4 10; 1 3 2), (1 9 15; 3 5 2),
 (5 12 13; 2 4 2), (6 7 15; 0 6 6), (1 4 14; 4 6 2), (2 8 10; 5 0 2), (3 9 11; 3 2 6),
 (9 12 15; 0 5 5), (2 6 14; 4 0 3), (3 4 10; 3 4 1), (8 11 13; 1 5 4), (1 5 7; 4 3 6),
 (1 6 7 11 15; 2 5 5 6 3 3 4 0 1 1), (2 4 8 12 13; 0 6 6 5 6 6 5 0 6 6), (3 5 9 10 14; 4 2 6 4 5 2 0 4 2 5),
 (3 5 8 11 14; 6 4 3 3 5 4 4 6 0), (1 4 7 12 13; 3 6 3 0 3 0 4 4 1 4), (2 6 9 10 15; 0 1 1 5 1 1 5 0 4 4).

Example A.17 A uniform $\{3, 5\}$ -LRGDD₇ of type 3^5 with $r_3 = 36$ and $r_5 = 3$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}, \{13, 14, 15\}\}$; each row forms a uniform parallel class:

(1 5 15; 6 3 4), (6 8 12; 3 0 4), (2 7 10; 2 6 4), (9 11 13; 1 3 2), (3 4 14; 1 5 4),
 (6 10 14; 2 3 1), (9 12 13; 3 2 6), (1 7 15; 0 1 1), (3 5 8; 4 0 3), (2 4 11; 4 0 3),
 (4 10 13; 3 0 4), (1 7 12; 4 5 1), (2 11 15; 4 3 6), (6 9 14; 6 2 3), (3 5 8; 3 5 2),
 (4 7 13; 2 2 0), (1 5 11; 0 2 2), (2 12 14; 4 6 2), (6 8 15; 2 4 2), (3 9 10; 5 4 6),
 (1 9 12; 4 4 0), (3 4 14; 5 0 2), (5 8 11; 1 3 2), (2 6 13; 3 0 4), (7 10 15; 6 0 1),
 (1 7 15; 3 2 6), (2 5 9; 6 0 1), (6 11 13; 1 0 6), (8 10 14; 5 3 5), (3 4 12; 6 4 5),
 (4 7 15; 3 0 4), (1 6 10; 3 6 3), (2 12 14; 2 3 1), (5 9 11; 2 6 4), (3 8 13; 4 2 5),
 (3 8 15; 6 5 6), (1 4 11; 3 3 0), (6 12 14; 2 5 3), (2 7 13; 6 2 3), (5 9 10; 5 0 2),
 (1 7 11; 5 5 0), (9 10 14; 3 6 3), (3 6 13; 0 3 3), (2 5 15; 5 4 6), (4 8 12; 2 2 0),
 (3 4 12; 2 1 6), (6 9 13; 1 2 1), (7 10 14; 1 0 6), (2 5 11; 0 1 1), (1 8 15; 4 5 1),
 (1 6 14; 4 5 1), (2 4 8; 2 1 6), (3 5 10; 0 6 6), (7 12 15; 0 3 3), (9 11 13; 6 0 1),
 (2 9 15; 3 5 2), (3 6 12; 5 3 5), (4 7 10; 1 4 3), (1 5 13; 4 5 1), (8 11 14; 3 0 4),
 (6 7 11; 5 0 2), (1 5 13; 1 4 3), (3 4 15; 3 2 6), (2 8 10; 2 2 0), (9 12 14; 5 2 4),
 (2 8 14; 4 2 5), (5 11 13; 0 4 4), (3 4 7; 0 6 6), (1 9 12; 0 2 2), (6 10 15; 0 0 0),
 (7 11 14; 1 3 2), (3 9 15; 2 1 6), (2 5 12; 3 0 4), (1 4 10; 0 5 5), (6 8 13; 6 1 2),
 (6 8 11; 5 6 1), (3 10 13; 1 4 3), (5 9 15; 3 1 5), (2 7 12; 1 6 5), (1 4 14; 2 3 1),
 (3 6 7; 2 4 2), (2 11 14; 2 1 6), (1 9 10; 2 3 1), (4 8 13; 1 5 4), (5 12 15; 0 2 2),
 (3 9 15; 3 3 0), (1 5 14; 2 1 6), (8 12 13; 1 1 0), (4 7 10; 4 2 5), (2 6 11; 2 5 3),
 (2 5 9; 4 1 4), (1 8 10; 0 2 2), (3 11 13; 3 1 5), (4 12 15; 0 4 4), (6 7 14; 6 4 5),
 (7 12 13; 2 4 2), (3 5 8; 6 3 4), (2 11 15; 6 0 1), (4 9 10; 3 0 4), (1 6 14; 0 0 0),
 (2 4 15; 6 1 2), (5 12 14; 2 2 0), (1 6 9; 1 3 2), (8 11 13; 0 3 3), (3 7 10; 5 5 0),
 (3 7 11; 1 6 5), (5 10 14; 4 1 4), (6 8 15; 0 3 3), (1 12 13; 3 1 5), (2 4 9; 3 4 1),
 (1 11 14; 4 4 0), (3 5 7; 5 0 2), (4 10 13; 1 3 2), (2 8 12; 6 1 2), (6 9 15; 5 6 1),
 (5 7 12; 3 6 3), (3 11 13; 5 5 0), (1 8 15; 3 0 4), (2 6 10; 5 4 6), (4 9 14; 6 6 0),
 (8 10 14; 1 1 0), (6 9 12; 3 4 1), (2 5 7; 1 0 6), (3 11 15; 0 0 0), (1 4 13; 5 6 1),
 (3 6 11; 4 2 5), (2 7 13; 5 6 1), (1 10 14; 4 6 2), (5 8 15; 5 3 5), (4 9 12; 4 1 4),
 (4 9 12; 5 4 6), (2 6 14; 1 0 6), (1 10 15; 0 6 6), (5 7 11; 0 4 4), (3 8 13; 1 0 6),
 (2 5 12; 2 5 3), (6 7 11; 1 4 3), (8 10 15; 4 0 3), (3 9 14; 1 2 1), (1 4 13; 4 3 6),
 (2 4 14; 0 5 5), (7 11 15; 6 2 3), (1 8 12; 5 1 3), (5 9 13; 0 5 5), (3 6 10; 1 2 1),
 (1 4 7; 6 6 0), (3 5 10; 1 3 2), (2 9 13; 5 4 6), (8 11 14; 4 2 5), (6 12 15; 1 2 1),
 (5 10 15; 3 5 2), (2 4 8; 5 3 5), (1 9 11; 5 1 3), (3 7 14; 2 1 6), (6 12 13; 3 6 3),
 (5 10 13; 1 0 6), (4 12 15; 3 1 5), (3 9 11; 4 4 0), (2 6 8; 6 0 1), (1 7 14; 1 2 1),
 (5 7 14; 5 0 2), (4 8 10; 3 6 3), (1 6 11; 5 0 2), (2 9 13; 6 3 4), (3 12 15; 6 6 0),
 (5 11 15; 5 0 2), (3 12 14; 5 3 5), (1 6 8; 2 6 4), (2 9 10; 2 0 5), (4 7 13; 5 4 6),
 (2 10 13; 3 1 5), (5 9 14; 6 3 4), (4 11 15; 1 5 4), (1 8 12; 1 0 6), (3 6 7; 6 3 4),
 (5 7 14; 1 5 4), (1 6 9; 6 6 0), (2 10 15; 1 6 5), (4 8 11; 0 6 6), (3 12 13; 2 6 4),
 (3 4 9 11 14; 4 6 1 4 2 4 0 2 5 3), (2 6 7 12 15; 4 4 3 2 0 6 5 6 5 6), (1 5 8 10 13; 3 2 1 2 6 5 6 6 0 1),
 (3 6 9 10 15; 3 0 0 4 4 4 1 0 4 4), (1 5 7 12 13; 5 2 6 0 4 1 2 4 5 1), (2 4 8 11 14; 1 5 3 4 4 2 3 5 6 1),
 (2 6 7 10 13; 0 3 5 5 3 5 5 2 2 0), (1 4 9 11 15; 1 1 6 4 0 5 3 5 3 5), (3 5 8 12 14; 2 2 0 6 0 5 4 5 4 6).

Example A.18 A uniform $\{3, 5\}$ -LRGDD₇ of type 3^5 with $r_3 = 34$ and $r_5 = 4$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}, \{13, 14, 15\}\}$; each row forms a uniform parallel class:

(1 5 13; 4 3 6), (3 8 11; 2 3 1), (6 9 10; 6 0 1), (2 12 15; 1 6 5), (4 7 14; 1 2 1),
 (7 12 15; 6 5 6), (4 11 13; 5 3 5), (2 9 10; 6 2 3), (1 5 14; 3 0 4), (3 6 8; 6 4 5),
 (3 5 9; 4 1 4), (4 7 11; 3 0 4), (6 10 15; 6 1 2), (2 12 13; 4 1 4), (1 8 14; 6 5 6),
 (3 4 9; 0 5 5), (6 10 14; 5 4 6), (1 11 13; 1 1 0), (2 8 15; 6 2 3), (5 7 12; 0 4 4),
 (4 9 14; 3 4 1), (2 7 15; 2 3 1), (1 6 12; 0 4 4), (3 11 13; 4 5 1), (5 8 10; 0 4 4),
 (3 11 14; 1 5 4), (5 9 13; 3 0 4), (4 8 12; 5 1 3), (2 7 10; 1 1 0), (1 6 15; 6 2 3),
 (3 6 8; 0 1 1), (1 10 15; 2 0 5), (2 7 13; 6 3 4), (4 12 14; 6 6 0), (5 9 11; 2 2 0),
 (5 12 15; 5 1 3), (1 6 8; 5 4 6), (2 4 10; 0 5 5), (3 7 13; 0 1 1), (9 11 14; 4 2 5),
 (6 11 13; 4 6 2), (3 5 7; 5 1 3), (4 9 14; 1 0 6), (2 10 15; 3 4 1), (1 8 12; 1 1 0),
 (3 4 7; 6 6 0), (5 10 14; 2 0 5), (2 12 15; 0 0 0), (1 6 9; 1 4 3), (8 11 13; 5 4 6),
 (3 9 12; 2 5 3), (2 6 14; 2 5 3), (1 7 13; 0 6 6), (5 11 15; 5 0 2), (4 8 10; 2 2 0),
 (3 4 10; 3 4 1), (6 9 12; 2 1 6), (7 11 14; 1 2 1), (1 8 15; 2 3 1), (2 5 13; 3 5 2),
 (9 11 15; 6 4 5), (2 6 8; 1 4 3), (5 7 10; 6 1 2), (3 12 13; 3 6 3), (1 4 14; 6 4 5),
 (8 10 13; 1 5 4), (2 5 14; 5 6 1), (3 6 7; 3 2 6), (1 4 12; 2 0 5), (9 11 15; 2 3 1),
 (6 10 14; 4 1 4), (5 9 11; 5 1 3), (2 4 15; 2 1 6), (1 7 12; 3 6 3), (3 8 13; 5 4 6),
 (2 6 9; 6 3 4), (3 4 13; 5 0 2), (5 7 12; 2 2 0), (1 11 15; 0 6 6), (8 10 14; 3 4 1),
 (2 4 8; 3 3 0), (3 10 14; 2 4 2), (6 7 12; 1 6 5), (9 11 15; 5 5 0), (1 5 13; 6 4 5),
 (9 12 13; 2 1 6), (2 5 8; 2 1 6), (1 4 11; 0 4 4), (7 10 15; 6 2 3), (3 6 14; 1 1 0),

$(3 \ 6 \ 7; 4 \ 4 \ 0), (2 \ 11 \ 13; 2 \ 6 \ 4), (4 \ 9 \ 10; 6 \ 4 \ 5), (1 \ 5 \ 15; 5 \ 1 \ 3), (8 \ 12 \ 14; 4 \ 5 \ 1),$
 $(6 \ 7 \ 15; 5 \ 5 \ 0), (3 \ 12 \ 14; 2 \ 0 \ 5), (1 \ 9 \ 10; 6 \ 5 \ 6), (2 \ 4 \ 11; 4 \ 3 \ 6), (5 \ 8 \ 13; 1 \ 4 \ 3),$
 $(2 \ 7 \ 11; 4 \ 4 \ 0), (3 \ 4 \ 15; 1 \ 2 \ 1), (8 \ 12 \ 14; 6 \ 3 \ 4), (1 \ 5 \ 9; 0 \ 1 \ 1), (6 \ 10 \ 13; 2 \ 0 \ 5),$
 $(3 \ 4 \ 12; 4 \ 1 \ 4), (9 \ 10 \ 13; 4 \ 6 \ 2), (5 \ 7 \ 15; 5 \ 2 \ 4), (1 \ 6 \ 11; 3 \ 6 \ 3), (2 \ 8 \ 14; 2 \ 3 \ 1),$
 $(4 \ 12 \ 15; 2 \ 3 \ 1), (2 \ 7 \ 10; 5 \ 6 \ 1), (1 \ 9 \ 14; 0 \ 3 \ 3), (3 \ 6 \ 11; 5 \ 6 \ 1), (5 \ 8 \ 13; 2 \ 3 \ 1),$
 $(3 \ 4 \ 13; 2 \ 2 \ 0), (6 \ 9 \ 12; 0 \ 5 \ 5), (8 \ 11 \ 15; 2 \ 5 \ 3), (1 \ 7 \ 10; 5 \ 1 \ 3), (2 \ 5 \ 14; 1 \ 4 \ 3),$
 $(1 \ 10 \ 13; 3 \ 2 \ 6), (6 \ 8 \ 15; 4 \ 4 \ 0), (2 \ 9 \ 11; 0 \ 1 \ 1), (3 \ 5 \ 12; 6 \ 6 \ 0), (4 \ 7 \ 14; 4 \ 1 \ 4),$
 $(6 \ 7 \ 13; 3 \ 1 \ 5), (1 \ 8 \ 12; 3 \ 5 \ 2), (5 \ 9 \ 14; 0 \ 5 \ 5), (3 \ 10 \ 15; 1 \ 0 \ 6), (2 \ 4 \ 11; 5 \ 0 \ 2),$
 $(4 \ 8 \ 15; 6 \ 5 \ 6), (7 \ 12 \ 13; 2 \ 3 \ 1), (2 \ 6 \ 11; 0 \ 6 \ 6), (3 \ 5 \ 10; 1 \ 0 \ 6), (1 \ 9 \ 14; 2 \ 6 \ 4),$
 $(1 \ 7 \ 14; 2 \ 1 \ 6), (6 \ 12 \ 13; 3 \ 3 \ 0), (3 \ 9 \ 15; 4 \ 3 \ 6), (2 \ 5 \ 10; 0 \ 0 \ 0), (4 \ 8 \ 11; 4 \ 1 \ 4),$
 $(6 \ 10 \ 15; 3 \ 0 \ 4), (3 \ 8 \ 11; 6 \ 5 \ 6), (1 \ 5 \ 12; 1 \ 2 \ 1), (4 \ 7 \ 13; 2 \ 4 \ 2), (2 \ 9 \ 14; 2 \ 2 \ 0),$
 $(3 \ 12 \ 14; 0 \ 3 \ 3), (2 \ 9 \ 13; 4 \ 2 \ 5), (1 \ 6 \ 15; 2 \ 4 \ 2), (4 \ 7 \ 11; 5 \ 3 \ 5), (5 \ 8 \ 10; 5 \ 3 \ 5),$
 $(1 \ 5 \ 11; 2 \ 5 \ 3), (3 \ 9 \ 10; 6 \ 6 \ 0), (7 \ 12 \ 14; 1 \ 0 \ 6), (6 \ 8 \ 13; 0 \ 2 \ 2), (2 \ 4 \ 15; 1 \ 5 \ 4),$
 $(5 \ 9 \ 15; 6 \ 6 \ 0), (2 \ 8 \ 12; 5 \ 6 \ 1), (1 \ 4 \ 7; 5 \ 4 \ 6), (6 \ 11 \ 13; 2 \ 5 \ 3), (3 \ 10 \ 14; 3 \ 6 \ 3),$
 $(1 \ 7 \ 10; 1 \ 6 \ 5), (4 \ 8 \ 15; 3 \ 0 \ 4), (2 \ 5 \ 12; 4 \ 3 \ 6), (3 \ 9 \ 13; 3 \ 3 \ 0), (6 \ 11 \ 14; 0 \ 6 \ 6),$
 $(3 \ 7 \ 15; 5 \ 4 \ 6), (2 \ 6 \ 9; 4 \ 5 \ 1), (4 \ 10 \ 14; 3 \ 3 \ 0), (5 \ 12 \ 13; 3 \ 1 \ 5), (1 \ 8 \ 11; 0 \ 3 \ 3),$
 $(2 \ 6 \ 8 \ 12 \ 14; 5 \ 0 \ 5 \ 0 \ 2 \ 0 \ 2 \ 5 \ 0 \ 2), (3 \ 5 \ 7 \ 11 \ 15; 2 \ 3 \ 2 \ 6 \ 1 \ 0 \ 4 \ 6 \ 3 \ 4), (1 \ 4 \ 9 \ 10 \ 13; 1 \ 5 \ 0 \ 0 \ 4 \ 6 \ 6 \ 2 \ 2 \ 0),$
 $(1 \ 4 \ 9 \ 12 \ 15; 3 \ 3 \ 3 \ 5 \ 0 \ 0 \ 2 \ 0 \ 2 \ 2), (3 \ 5 \ 8 \ 11 \ 14; 3 \ 0 \ 0 \ 2 \ 4 \ 4 \ 6 \ 0 \ 2 \ 2), (2 \ 6 \ 7 \ 10 \ 13; 3 \ 0 \ 4 \ 0 \ 4 \ 1 \ 4 \ 4 \ 0 \ 3),$
 $(2 \ 5 \ 7 \ 11 \ 14; 6 \ 3 \ 5 \ 1 \ 4 \ 6 \ 2 \ 2 \ 5 \ 3), (1 \ 4 \ 8 \ 10 \ 13; 4 \ 5 \ 4 \ 5 \ 1 \ 0 \ 1 \ 6 \ 0 \ 1), (3 \ 6 \ 9 \ 12 \ 15; 2 \ 0 \ 4 \ 1 \ 5 \ 2 \ 6 \ 4 \ 1 \ 4),$
 $(2 \ 4 \ 9 \ 12 \ 13; 6 \ 1 \ 2 \ 4 \ 2 \ 3 \ 5 \ 1 \ 3 \ 2), (1 \ 6 \ 7 \ 11 \ 14; 4 \ 6 \ 2 \ 2 \ 2 \ 5 \ 5 \ 3 \ 3 \ 0), (3 \ 5 \ 8 \ 10 \ 15; 0 \ 3 \ 5 \ 5 \ 3 \ 5 \ 5 \ 2 \ 2 \ 0).$

Example A.19 A uniform $\{3, 5\}$ -LRGDD₇ of type 3^5 with $r_3 = 32$ and $r_5 = 5$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}, \{13, 14, 15\}\}$; each row forms a uniform parallel class:

$(3 \ 9 \ 14; 6 \ 1 \ 2), (8 \ 12 \ 15; 6 \ 3 \ 4), (5 \ 11 \ 13; 5 \ 4 \ 6), (2 \ 4 \ 7; 6 \ 2 \ 3), (1 \ 6 \ 10; 1 \ 5 \ 4),$
 $(6 \ 8 \ 15; 5 \ 2 \ 4), (3 \ 11 \ 14; 3 \ 3 \ 0), (4 \ 9 \ 10; 4 \ 1 \ 4), (2 \ 7 \ 13; 4 \ 1 \ 4), (1 \ 5 \ 12; 1 \ 0 \ 6),$
 $(6 \ 10 \ 14; 5 \ 2 \ 4), (2 \ 4 \ 15; 5 \ 2 \ 4), (5 \ 9 \ 11; 5 \ 6 \ 1), (3 \ 8 \ 12; 3 \ 6 \ 3), (1 \ 7 \ 13; 0 \ 6 \ 6),$
 $(2 \ 4 \ 9; 4 \ 4 \ 0), (6 \ 7 \ 11; 0 \ 4 \ 4), (3 \ 8 \ 13; 4 \ 2 \ 5), (1 \ 12 \ 15; 2 \ 0 \ 5), (5 \ 10 \ 14; 3 \ 3 \ 0),$
 $(4 \ 7 \ 10; 1 \ 4 \ 3), (2 \ 6 \ 15; 3 \ 1 \ 5), (3 \ 9 \ 12; 1 \ 4 \ 3), (5 \ 11 \ 13; 1 \ 6 \ 5), (1 \ 8 \ 14; 2 \ 3 \ 1),$
 $(1 \ 6 \ 12; 4 \ 1 \ 4), (9 \ 10 \ 13; 6 \ 1 \ 2), (2 \ 7 \ 11; 3 \ 2 \ 6), (3 \ 4 \ 14; 5 \ 4 \ 6), (5 \ 8 \ 15; 6 \ 0 \ 1),$
 $(1 \ 4 \ 10; 3 \ 3 \ 0), (3 \ 11 \ 15; 6 \ 3 \ 4), (6 \ 9 \ 14; 6 \ 4 \ 5), (7 \ 12 \ 13; 1 \ 1 \ 0), (2 \ 5 \ 8; 1 \ 4 \ 3),$
 $(2 \ 9 \ 10; 3 \ 4 \ 1), (3 \ 4 \ 15; 6 \ 0 \ 1), (6 \ 7 \ 11; 4 \ 6 \ 2), (5 \ 12 \ 14; 2 \ 0 \ 5), (1 \ 8 \ 13; 6 \ 0 \ 1),$
 $(4 \ 10 \ 13; 2 \ 5 \ 3), (2 \ 6 \ 8; 4 \ 5 \ 1), (1 \ 7 \ 14; 5 \ 4 \ 6), (5 \ 9 \ 11; 0 \ 0 \ 0), (3 \ 12 \ 15; 3 \ 2 \ 6),$
 $(6 \ 7 \ 14; 1 \ 1 \ 0), (9 \ 12 \ 13; 6 \ 3 \ 4), (1 \ 5 \ 10; 4 \ 4 \ 0), (3 \ 4 \ 15; 1 \ 6 \ 5), (2 \ 8 \ 11; 3 \ 1 \ 5),$
 $(4 \ 7 \ 13; 5 \ 3 \ 5), (2 \ 10 \ 14; 5 \ 1 \ 3), (1 \ 5 \ 12; 0 \ 3 \ 3), (3 \ 6 \ 8; 4 \ 6 \ 2), (9 \ 11 \ 15; 4 \ 4 \ 0),$
 $(8 \ 10 \ 14; 4 \ 2 \ 5), (1 \ 7 \ 15; 3 \ 3 \ 0), (2 \ 6 \ 9; 6 \ 1 \ 2), (3 \ 4 \ 11; 0 \ 2 \ 2), (5 \ 12 \ 13; 5 \ 0 \ 2),$
 $(2 \ 10 \ 14; 0 \ 2 \ 2), (5 \ 9 \ 15; 6 \ 2 \ 3), (1 \ 6 \ 13; 3 \ 3 \ 0), (4 \ 7 \ 11; 0 \ 0 \ 0), (3 \ 8 \ 12; 2 \ 2 \ 0),$
 $(4 \ 8 \ 14; 5 \ 5 \ 0), (7 \ 10 \ 15; 5 \ 4 \ 6), (2 \ 6 \ 12; 1 \ 3 \ 2), (1 \ 9 \ 13; 0 \ 4 \ 4), (3 \ 5 \ 11; 5 \ 1 \ 3),$
 $(6 \ 9 \ 14; 4 \ 3 \ 6), (3 \ 7 \ 10; 3 \ 3 \ 0), (4 \ 12 \ 15; 6 \ 0 \ 1), (1 \ 8 \ 11; 0 \ 6 \ 6), (2 \ 5 \ 13; 3 \ 6 \ 3),$
 $(8 \ 12 \ 14; 2 \ 4 \ 2), (2 \ 4 \ 9; 0 \ 6 \ 6), (6 \ 7 \ 11; 6 \ 0 \ 1), (3 \ 5 \ 13; 6 \ 0 \ 1), (1 \ 10 \ 15; 6 \ 1 \ 2),$
 $(2 \ 10 \ 14; 6 \ 0 \ 1), (3 \ 6 \ 8; 3 \ 0 \ 4), (1 \ 11 \ 13; 5 \ 5 \ 0), (5 \ 7 \ 15; 0 \ 1 \ 1), (4 \ 9 \ 12; 2 \ 0 \ 5),$
 $(3 \ 11 \ 13; 4 \ 6 \ 2), (2 \ 9 \ 12; 2 \ 4 \ 2), (5 \ 7 \ 14; 5 \ 2 \ 4), (1 \ 6 \ 10; 0 \ 0 \ 0), (4 \ 8 \ 15; 0 \ 2 \ 2),$
 $(4 \ 10 \ 15; 5 \ 3 \ 5), (3 \ 5 \ 7; 1 \ 2 \ 1), (2 \ 11 \ 13; 4 \ 5 \ 1), (6 \ 8 \ 14; 3 \ 6 \ 3), (1 \ 9 \ 12; 6 \ 6 \ 0),$
 $(4 \ 9 \ 13; 1 \ 0 \ 6), (5 \ 12 \ 14; 0 \ 4 \ 4), (2 \ 7 \ 15; 5 \ 3 \ 5), (1 \ 8 \ 10; 5 \ 1 \ 3), (3 \ 6 \ 11; 2 \ 5 \ 3),$
 $(3 \ 9 \ 14; 2 \ 2 \ 0), (1 \ 6 \ 15; 5 \ 6 \ 1), (2 \ 5 \ 12; 4 \ 1 \ 4), (4 \ 8 \ 11; 4 \ 5 \ 1), (7 \ 10 \ 13; 1 \ 2 \ 1),$
 $(3 \ 6 \ 15; 1 \ 1 \ 0), (1 \ 7 \ 13; 6 \ 2 \ 3), (2 \ 11 \ 14; 6 \ 4 \ 5), (5 \ 9 \ 10; 1 \ 4 \ 3), (4 \ 8 \ 12; 3 \ 1 \ 5),$
 $(3 \ 6 \ 9; 0 \ 3 \ 3), (1 \ 4 \ 11; 2 \ 1 \ 6), (7 \ 12 \ 14; 3 \ 2 \ 6), (8 \ 10 \ 15; 0 \ 0 \ 0), (2 \ 5 \ 13; 5 \ 0 \ 2),$
 $(3 \ 7 \ 10; 0 \ 6 \ 6), (9 \ 11 \ 15; 5 \ 6 \ 1), (2 \ 5 \ 8; 6 \ 6 \ 0), (6 \ 12 \ 13; 6 \ 5 \ 6), (1 \ 4 \ 14; 1 \ 5 \ 4),$
 $(6 \ 12 \ 14; 5 \ 5 \ 0), (2 \ 5 \ 8; 2 \ 0 \ 5), (3 \ 9 \ 10; 0 \ 5 \ 5), (4 \ 11 \ 13; 1 \ 4 \ 3), (1 \ 7 \ 15; 1 \ 4 \ 3),$
 $(6 \ 7 \ 12; 3 \ 1 \ 5), (3 \ 8 \ 10; 1 \ 0 \ 6), (2 \ 9 \ 13; 5 \ 3 \ 5), (4 \ 11 \ 15; 4 \ 6 \ 2), (1 \ 5 \ 14; 5 \ 6 \ 1),$
 $(5 \ 9 \ 10; 2 \ 2 \ 0), (8 \ 11 \ 14; 4 \ 5 \ 1), (1 \ 4 \ 7; 5 \ 2 \ 4), (3 \ 6 \ 13; 6 \ 3 \ 4), (2 \ 12 \ 15; 0 \ 0 \ 0),$
 $(6 \ 10 \ 13; 6 \ 3 \ 4), (1 \ 5 \ 8; 3 \ 4 \ 1), (9 \ 11 \ 15; 2 \ 5 \ 3), (2 \ 7 \ 12; 1 \ 5 \ 4), (3 \ 4 \ 14; 4 \ 5 \ 1),$
 $(2 \ 4 \ 11; 2 \ 5 \ 3), (6 \ 7 \ 12; 5 \ 0 \ 2), (5 \ 8 \ 13; 2 \ 5 \ 3), (3 \ 10 \ 15; 4 \ 5 \ 1), (1 \ 9 \ 14; 1 \ 2 \ 1),$
 $(5 \ 7 \ 15; 4 \ 6 \ 2), (2 \ 11 \ 14; 0 \ 6 \ 6), (4 \ 8 \ 10; 1 \ 6 \ 5), (3 \ 9 \ 13; 5 \ 5 \ 0), (1 \ 6 \ 12; 2 \ 5 \ 3),$
 $(6 \ 9 \ 15; 1 \ 3 \ 2), (2 \ 4 \ 14; 1 \ 3 \ 2), (3 \ 5 \ 10; 4 \ 2 \ 5), (1 \ 7 \ 11; 4 \ 2 \ 5), (8 \ 12 \ 13; 1 \ 4 \ 3),$
 $(6 \ 10 \ 13; 3 \ 1 \ 5), (3 \ 5 \ 7; 3 \ 6 \ 3), (8 \ 11 \ 14; 3 \ 6 \ 3), (2 \ 12 \ 15; 2 \ 4 \ 2), (1 \ 4 \ 9; 4 \ 2 \ 5),$
 $(1 \ 4 \ 8 \ 10 \ 13; 6 \ 1 \ 2 \ 1 \ 2 \ 3 \ 2 \ 1 \ 0 \ 6), (3 \ 5 \ 9 \ 12 \ 15; 0 \ 4 \ 1 \ 4 \ 4 \ 1 \ 4 \ 4 \ 0 \ 3), (2 \ 6 \ 7 \ 11 \ 14; 5 \ 0 \ 3 \ 5 \ 2 \ 5 \ 0 \ 3 \ 5 \ 2),$
 $(3 \ 4 \ 7 \ 12 \ 13; 2 \ 1 \ 0 \ 1 \ 6 \ 5 \ 6 \ 0 \ 1), (1 \ 5 \ 9 \ 11 \ 14; 2 \ 5 \ 4 \ 1 \ 3 \ 2 \ 6 \ 6 \ 3 \ 4), (2 \ 6 \ 8 \ 10 \ 15; 2 \ 1 \ 3 \ 6 \ 6 \ 1 \ 4 \ 2 \ 5 \ 3),$
 $(1 \ 5 \ 8 \ 11 \ 15; 6 \ 3 \ 3 \ 2 \ 4 \ 4 \ 3 \ 0 \ 6 \ 6), (2 \ 6 \ 9 \ 10 \ 13; 0 \ 0 \ 2 \ 2 \ 0 \ 2 \ 2 \ 2 \ 2 \ 0), (3 \ 4 \ 7 \ 12 \ 14; 3 \ 5 \ 5 \ 6 \ 2 \ 2 \ 3 \ 0 \ 1 \ 1),$
 $(1 \ 6 \ 9 \ 11 \ 15; 6 \ 4 \ 0 \ 5 \ 5 \ 1 \ 6 \ 3 \ 1 \ 5), (2 \ 4 \ 8 \ 12 \ 13; 3 \ 2 \ 6 \ 4 \ 6 \ 3 \ 1 \ 4 \ 2 \ 5), (3 \ 5 \ 7 \ 10 \ 14; 2 \ 4 \ 1 \ 0 \ 2 \ 6 \ 5 \ 4 \ 3 \ 6),$
 $(3 \ 6 \ 8 \ 11 \ 13; 5 \ 5 \ 0 \ 4 \ 0 \ 2 \ 6 \ 2 \ 6 \ 4), (2 \ 5 \ 7 \ 10 \ 15; 0 \ 6 \ 1 \ 5 \ 6 \ 1 \ 5 \ 2 \ 6 \ 4), (1 \ 4 \ 9 \ 12 \ 14; 0 \ 3 \ 4 \ 0 \ 3 \ 4 \ 0 \ 1 \ 4 \ 3).$

Example A.20 A uniform $\{3, 4\}$ -LRGDD₄ of type 3^4 with $r_3 = 15$ and $r_4 = 2$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}\}$; each row forms a uniform parallel class:

$(2\ 4\ 7; 0\ 2\ 2)$, $(3\ 5\ 12; 1\ 2\ 1)$, $(6\ 9\ 11; 0\ 1\ 1)$, $(1\ 8\ 10; 3\ 0\ 1)$,
 $(2\ 5\ 12; 0\ 0\ 0)$, $(1\ 8\ 11; 1\ 0\ 3)$, $(3\ 6\ 9; 1\ 0\ 3)$, $(4\ 7\ 10; 3\ 2\ 3)$,
 $(2\ 4\ 11; 1\ 1\ 0)$, $(3\ 5\ 9; 3\ 3\ 0)$, $(6\ 8\ 10; 1\ 0\ 3)$, $(1\ 7\ 12; 2\ 2\ 0)$,
 $(2\ 5\ 10; 2\ 2\ 0)$, $(3\ 9\ 11; 1\ 1\ 0)$, $(4\ 8\ 12; 0\ 3\ 3)$, $(1\ 6\ 7; 0\ 3\ 3)$,
 $(2\ 9\ 12; 3\ 1\ 2)$, $(1\ 4\ 10; 2\ 3\ 1)$, $(3\ 6\ 8; 3\ 3\ 0)$, $(5\ 7\ 11; 3\ 2\ 3)$,
 $(1\ 5\ 10; 0\ 1\ 1)$, $(2\ 6\ 8; 2\ 1\ 3)$, $(3\ 7\ 12; 1\ 3\ 2)$, $(4\ 9\ 11; 0\ 3\ 3)$,
 $(1\ 4\ 9; 3\ 2\ 3)$, $(2\ 5\ 11; 1\ 2\ 1)$, $(3\ 7\ 10; 0\ 2\ 2)$, $(6\ 8\ 12; 2\ 2\ 0)$,
 $(2\ 8\ 11; 3\ 0\ 1)$, $(3\ 4\ 7; 3\ 3\ 0)$, $(5\ 9\ 12; 1\ 2\ 1)$, $(1\ 6\ 10; 3\ 2\ 3)$,
 $(2\ 6\ 11; 3\ 3\ 0)$, $(1\ 5\ 7; 2\ 0\ 2)$, $(3\ 9\ 10; 2\ 3\ 1)$, $(4\ 8\ 12; 2\ 0\ 2)$,
 $(3\ 4\ 11; 1\ 3\ 2)$, $(2\ 5\ 8; 3\ 0\ 1)$, $(6\ 7\ 10; 1\ 2\ 1)$, $(1\ 9\ 12; 0\ 3\ 3)$,
 $(2\ 7\ 10; 0\ 0\ 0)$, $(1\ 5\ 8; 1\ 0\ 3)$, $(3\ 6\ 11; 2\ 0\ 2)$, $(4\ 9\ 12; 2\ 2\ 0)$,
 $(3\ 5\ 8; 0\ 2\ 2)$, $(1\ 4\ 12; 0\ 1\ 1)$, $(6\ 7\ 11; 2\ 3\ 1)$, $(2\ 9\ 10; 1\ 3\ 2)$,
 $(2\ 8\ 12; 2\ 3\ 1)$, $(1\ 6\ 9; 2\ 3\ 1)$, $(3\ 4\ 10; 0\ 0\ 0)$, $(5\ 7\ 11; 1\ 3\ 2)$,
 $(3\ 6\ 12; 0\ 0\ 0)$, $(2\ 4\ 9; 3\ 0\ 1)$, $(1\ 7\ 11; 1\ 1\ 0)$, $(5\ 8\ 10; 0\ 2\ 2)$,
 $(2\ 4\ 7; 2\ 3\ 1)$, $(1\ 6\ 12; 1\ 0\ 3)$, $(3\ 8\ 11; 0\ 2\ 2)$, $(5\ 9\ 10; 3\ 3\ 0)$,
 $(1\ 5\ 9\ 11; 3\ 1\ 3\ 2\ 0\ 2)$, $(2\ 6\ 7\ 12; 1\ 1\ 2\ 0\ 1\ 1)$, $(3\ 4\ 8\ 10; 2\ 1\ 1\ 3\ 3\ 0)$,
 $(1\ 4\ 8\ 11; 1\ 2\ 2\ 1\ 1\ 0)$, $(2\ 6\ 9\ 10; 0\ 2\ 1\ 2\ 1\ 3)$, $(3\ 5\ 7\ 12; 2\ 2\ 1\ 0\ 3\ 3)$.

Example A.21 A uniform $\{3, 4\}$ -LRGDD₄ of type 3^4 with $r_3 = 12$ and $r_4 = 4$,

$\mathbf{G} = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}\}$; each row forms a uniform parallel class:

$(1\ 6\ 12; 2\ 1\ 3)$, $(2\ 8\ 11; 3\ 0\ 1)$, $(5\ 7\ 10; 3\ 1\ 2)$, $(3\ 4\ 9; 1\ 3\ 2)$,
 $(1\ 4\ 12; 0\ 0\ 0)$, $(3\ 5\ 8; 1\ 0\ 3)$, $(6\ 7\ 10; 3\ 0\ 1)$, $(2\ 9\ 11; 0\ 2\ 2)$,
 $(3\ 8\ 11; 1\ 0\ 3)$, $(2\ 5\ 10; 2\ 0\ 2)$, $(6\ 9\ 12; 1\ 2\ 1)$, $(1\ 4\ 7; 1\ 1\ 0)$,
 $(6\ 9\ 11; 3\ 2\ 3)$, $(1\ 4\ 10; 3\ 0\ 1)$, $(2\ 5\ 8; 0\ 2\ 2)$, $(3\ 7\ 12; 0\ 3\ 3)$,
 $(6\ 8\ 11; 2\ 0\ 2)$, $(2\ 4\ 9; 2\ 2\ 0)$, $(1\ 5\ 10; 1\ 1\ 0)$, $(3\ 7\ 12; 1\ 2\ 1)$,
 $(1\ 4\ 8; 2\ 2\ 0)$, $(3\ 7\ 12; 2\ 0\ 2)$, $(2\ 6\ 10; 1\ 2\ 1)$, $(5\ 9\ 11; 1\ 1\ 0)$,
 $(2\ 6\ 7; 2\ 3\ 1)$, $(3\ 4\ 11; 3\ 1\ 2)$, $(5\ 9\ 12; 2\ 0\ 2)$, $(1\ 8\ 10; 3\ 2\ 3)$,
 $(1\ 9\ 12; 2\ 2\ 0)$, $(4\ 7\ 10; 2\ 2\ 0)$, $(3\ 6\ 8; 3\ 3\ 0)$, $(2\ 5\ 11; 1\ 1\ 0)$,
 $(2\ 6\ 12; 3\ 3\ 0)$, $(4\ 8\ 10; 1\ 3\ 2)$, $(3\ 5\ 9; 2\ 2\ 0)$, $(1\ 7\ 11; 2\ 3\ 1)$,
 $(2\ 8\ 10; 1\ 1\ 0)$, $(4\ 7\ 11; 3\ 3\ 0)$, $(3\ 6\ 12; 0\ 1\ 1)$, $(1\ 5\ 9; 2\ 1\ 3)$,
 $(4\ 8\ 12; 2\ 1\ 3)$, $(2\ 6\ 7; 0\ 2\ 2)$, $(3\ 5\ 11; 0\ 2\ 2)$, $(1\ 9\ 10; 3\ 3\ 0)$,
 $(3\ 4\ 11; 2\ 3\ 1)$, $(1\ 6\ 7; 3\ 3\ 0)$, $(2\ 9\ 10; 1\ 3\ 2)$, $(5\ 8\ 12; 0\ 1\ 1)$,
 $(1\ 5\ 7\ 11; 3\ 0\ 2\ 1\ 3\ 2)$, $(2\ 4\ 9\ 12; 0\ 3\ 2\ 3\ 2\ 3)$, $(3\ 6\ 8\ 10; 1\ 2\ 3\ 1\ 2\ 1)$,
 $(3\ 6\ 9\ 10; 2\ 0\ 1\ 2\ 3\ 1)$, $(2\ 4\ 7\ 11; 3\ 0\ 3\ 1\ 0\ 3)$, $(1\ 5\ 8\ 12; 0\ 1\ 3\ 1\ 3\ 2)$,
 $(2\ 4\ 8\ 12; 1\ 0\ 0\ 3\ 3\ 0)$, $(3\ 5\ 7\ 10; 3\ 3\ 2\ 0\ 3\ 3)$, $(1\ 6\ 9\ 11; 0\ 0\ 1\ 0\ 1\ 1)$,
 $(3\ 4\ 9\ 10; 0\ 1\ 0\ 1\ 0\ 3)$, $(2\ 5\ 7\ 12; 3\ 1\ 1\ 2\ 2\ 0)$, $(1\ 6\ 8\ 11; 1\ 0\ 0\ 3\ 3\ 0)$.

Example A.22 A uniform $\{3, 4\}$ -LRGDD₄ of type 3^4 with $r_3 = 9$ and $r_4 = 6$,

$\mathbf{G} = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}\}$; each row forms a uniform parallel class:

$(2\ 4\ 7; 1\ 3\ 2)$, $(3\ 9\ 11; 0\ 1\ 1)$, $(1\ 5\ 10; 3\ 1\ 2)$, $(6\ 8\ 12; 2\ 0\ 2)$,
 $(1\ 8\ 12; 2\ 3\ 1)$, $(6\ 7\ 11; 1\ 2\ 1)$, $(3\ 4\ 10; 2\ 0\ 2)$, $(2\ 5\ 9; 2\ 2\ 0)$,
 $(5\ 8\ 11; 3\ 0\ 1)$, $(2\ 7\ 10; 0\ 1\ 1)$, $(1\ 6\ 12; 1\ 2\ 1)$, $(3\ 4\ 9; 1\ 2\ 1)$,
 $(2\ 8\ 11; 3\ 2\ 3)$, $(3\ 5\ 12; 3\ 1\ 2)$, $(4\ 9\ 10; 2\ 3\ 1)$, $(1\ 6\ 7; 2\ 0\ 2)$,
 $(6\ 9\ 11; 1\ 0\ 3)$, $(1\ 5\ 7; 2\ 1\ 3)$, $(2\ 4\ 12; 2\ 3\ 1)$, $(3\ 8\ 10; 1\ 1\ 0)$,
 $(4\ 7\ 12; 0\ 0\ 0)$, $(3\ 5\ 11; 1\ 0\ 3)$, $(2\ 6\ 8; 1\ 2\ 1)$, $(1\ 9\ 10; 0\ 3\ 3)$,
 $(2\ 6\ 10; 2\ 3\ 1)$, $(4\ 8\ 12; 2\ 2\ 0)$, $(3\ 7\ 11; 0\ 3\ 3)$, $(1\ 5\ 9; 0\ 3\ 3)$,
 $(1\ 7\ 12; 3\ 1\ 2)$, $(2\ 6\ 11; 0\ 3\ 3)$, $(5\ 9\ 10; 1\ 1\ 0)$, $(3\ 4\ 8; 0\ 3\ 3)$,
 $(5\ 7\ 10; 2\ 0\ 2)$, $(2\ 9\ 12; 0\ 0\ 0)$, $(1\ 4\ 11; 3\ 3\ 0)$, $(3\ 6\ 8; 0\ 0\ 0)$,
 $(3\ 6\ 7\ 12; 2\ 1\ 0\ 3\ 2\ 3)$, $(2\ 5\ 8\ 10; 3\ 0\ 2\ 1\ 3\ 2)$, $(1\ 4\ 9\ 11; 2\ 1\ 1\ 3\ 3\ 0)$,
 $(2\ 5\ 9\ 12; 1\ 3\ 1\ 2\ 0\ 2)$, $(1\ 4\ 8\ 11; 0\ 0\ 2\ 0\ 2\ 2)$, $(3\ 6\ 7\ 10; 3\ 3\ 2\ 0\ 3\ 3)$,

$(1\ 5\ 8\ 12; 1\ 1\ 0\ 0\ 3\ 3), (3\ 6\ 9\ 10; 1\ 1\ 3\ 0\ 2\ 2), (2\ 4\ 7\ 11; 3\ 2\ 0\ 3\ 1\ 2),$
 $(1\ 6\ 8\ 10; 0\ 3\ 0\ 3\ 0\ 1), (2\ 5\ 7\ 11; 0\ 1\ 1\ 1\ 1\ 0), (3\ 4\ 9\ 12; 3\ 3\ 2\ 0\ 3\ 3),$
 $(3\ 5\ 7\ 12; 2\ 2\ 3\ 0\ 1\ 1), (1\ 6\ 9\ 11; 3\ 2\ 0\ 3\ 1\ 2), (2\ 4\ 8\ 10; 0\ 1\ 0\ 1\ 0\ 3),$
 $(2\ 6\ 9\ 12; 3\ 1\ 2\ 2\ 3\ 1), (3\ 5\ 8\ 11; 0\ 2\ 2\ 2\ 2\ 0), (1\ 4\ 7\ 10; 1\ 2\ 2\ 1\ 1\ 0).$

Example A.23 A uniform $\{3, 4\}$ -LRGDD₄ of type 3^4 with $r_3 = 6$ and $r_4 = 8$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}\}$; each row forms a uniform parallel class:

$(3\ 8\ 11; 2\ 1\ 3), (4\ 9\ 10; 0\ 1\ 1), (1\ 5\ 7; 2\ 3\ 1), (2\ 6\ 12; 1\ 3\ 2),$
 $(3\ 4\ 8; 3\ 0\ 1), (5\ 9\ 12; 2\ 3\ 1), (1\ 6\ 11; 3\ 0\ 1), (2\ 7\ 10; 0\ 2\ 2),$
 $(3\ 4\ 12; 1\ 1\ 0), (6\ 7\ 11; 0\ 0\ 0), (2\ 5\ 9; 2\ 3\ 1), (1\ 8\ 10; 1\ 3\ 2),$
 $(1\ 7\ 12; 2\ 0\ 2), (2\ 6\ 9; 2\ 1\ 3), (5\ 8\ 10; 1\ 1\ 0), (3\ 4\ 11; 0\ 0\ 0),$
 $(3\ 9\ 11; 3\ 2\ 3), (2\ 5\ 10; 0\ 0\ 0), (1\ 4\ 7; 1\ 0\ 3), (6\ 8\ 12; 0\ 3\ 3),$
 $(2\ 9\ 12; 0\ 2\ 2), (3\ 6\ 8; 0\ 1\ 1), (4\ 7\ 11; 1\ 2\ 1), (1\ 5\ 10; 1\ 0\ 3),$
 $(1\ 6\ 8\ 10; 1\ 0\ 1\ 3\ 0\ 1), (3\ 5\ 9\ 12; 2\ 2\ 2\ 0\ 0\ 0), (2\ 4\ 7\ 11; 1\ 3\ 2\ 2\ 1\ 3),$
 $(3\ 6\ 9\ 10; 1\ 1\ 3\ 0\ 2\ 2), (2\ 4\ 7\ 12; 2\ 2\ 1\ 0\ 3\ 3), (1\ 5\ 8\ 11; 0\ 3\ 1\ 3\ 1\ 2),$
 $(2\ 6\ 7\ 10; 0\ 1\ 1\ 1\ 1\ 0), (3\ 5\ 8\ 12; 3\ 3\ 0\ 0\ 1\ 1), (1\ 4\ 9\ 11; 3\ 2\ 2\ 3\ 3\ 0),$
 $(3\ 4\ 9\ 10; 2\ 0\ 0\ 2\ 2\ 0), (1\ 5\ 7\ 12; 3\ 1\ 1\ 2\ 2\ 0), (2\ 6\ 8\ 11; 3\ 1\ 1\ 2\ 2\ 0),$
 $(1\ 4\ 8\ 12; 0\ 2\ 2\ 2\ 2\ 0), (3\ 6\ 7\ 10; 2\ 0\ 1\ 2\ 3\ 1), (2\ 5\ 9\ 11; 3\ 2\ 3\ 3\ 0\ 1),$
 $(3\ 6\ 7\ 12; 3\ 2\ 3\ 3\ 0\ 1), (1\ 4\ 9\ 10; 2\ 3\ 2\ 1\ 0\ 3), (2\ 5\ 8\ 11; 1\ 3\ 0\ 2\ 3\ 1),$
 $(1\ 6\ 9\ 12; 2\ 0\ 3\ 2\ 1\ 3), (3\ 5\ 7\ 11; 1\ 1\ 3\ 0\ 2\ 2), (2\ 4\ 8\ 10; 0\ 0\ 3\ 0\ 3\ 3),$
 $(1\ 6\ 9\ 11; 0\ 1\ 3\ 1\ 3\ 2), (3\ 5\ 7\ 10; 0\ 3\ 2\ 3\ 2\ 3), (2\ 4\ 8\ 12; 3\ 2\ 0\ 3\ 1\ 2).$

Example A.24 A uniform $\{3, 4\}$ -LRGDD₁₂ of type 3^4 with $r_3 = 18$ and $r_4 = 24$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}\}$; each row forms a uniform parallel class:

$(5\ 9\ 12; 4\ 0\ 8), (3\ 6\ 11; 5\ 10\ 5), (1\ 4\ 7; 7\ 7\ 0), (2\ 8\ 10; 4\ 5\ 1),$
 $(3\ 4\ 12; 4\ 1\ 9), (2\ 6\ 7; 10\ 5\ 7), (1\ 8\ 10; 1\ 3\ 2), (5\ 9\ 11; 6\ 4\ 10),$
 $(5\ 7\ 12; 11\ 5\ 6), (1\ 4\ 11; 9\ 1\ 4), (2\ 6\ 8; 0\ 8\ 8), (3\ 9\ 10; 11\ 6\ 7),$
 $(2\ 6\ 10; 4\ 7\ 3), (3\ 8\ 12; 4\ 10\ 6), (4\ 9\ 11; 5\ 11\ 6), (1\ 5\ 7; 2\ 8\ 6),$
 $(4\ 9\ 12; 0\ 6\ 6), (1\ 8\ 11; 10\ 6\ 8), (2\ 5\ 7; 4\ 6\ 2), (3\ 6\ 10; 11\ 0\ 1),$
 $(4\ 8\ 10; 7\ 3\ 8), (1\ 5\ 9; 5\ 6\ 1), (2\ 7\ 12; 1\ 2\ 1), (3\ 6\ 11; 3\ 2\ 11),$
 $(2\ 7\ 11; 0\ 10\ 10), (5\ 8\ 10; 3\ 6\ 3), (1\ 4\ 12; 10\ 9\ 11), (3\ 6\ 9; 8\ 0\ 4),$
 $(4\ 7\ 11; 6\ 7\ 1), (3\ 5\ 9; 11\ 8\ 9), (1\ 6\ 10; 7\ 9\ 2), (2\ 8\ 12; 10\ 10\ 0),$
 $(1\ 5\ 12; 9\ 10\ 1), (3\ 7\ 10; 5\ 2\ 9), (2\ 4\ 9; 8\ 4\ 8), (6\ 8\ 11; 5\ 7\ 2),$
 $(2\ 5\ 11; 10\ 11\ 1), (1\ 9\ 12; 3\ 7\ 4), (3\ 4\ 7; 10\ 0\ 2), (6\ 8\ 10; 1\ 0\ 11),$
 $(1\ 4\ 8; 2\ 8\ 6), (2\ 7\ 11; 7\ 4\ 9), (6\ 9\ 10; 11\ 4\ 5), (3\ 5\ 12; 0\ 11\ 11),$
 $(2\ 6\ 12; 8\ 5\ 9), (3\ 4\ 8; 8\ 11\ 3), (5\ 7\ 10; 9\ 4\ 7), (1\ 9\ 11; 4\ 8\ 4),$
 $(3\ 9\ 12; 1\ 8\ 7), (2\ 5\ 8; 9\ 7\ 10), (1\ 4\ 10; 3\ 5\ 2), (6\ 7\ 11; 2\ 8\ 6),$
 $(2\ 6\ 9; 3\ 0\ 9), (5\ 8\ 11; 2\ 5\ 3), (3\ 7\ 12; 4\ 0\ 8), (1\ 4\ 10; 8\ 2\ 6),$
 $(2\ 5\ 11; 1\ 9\ 8), (3\ 4\ 8; 9\ 5\ 8), (6\ 9\ 12; 0\ 5\ 5), (1\ 7\ 10; 10\ 8\ 10),$
 $(3\ 5\ 11; 5\ 3\ 10), (2\ 9\ 10; 9\ 3\ 6), (1\ 6\ 8; 3\ 0\ 9), (4\ 7\ 12; 4\ 8\ 4),$
 $(1\ 5\ 9; 4\ 7\ 3), (3\ 8\ 11; 6\ 1\ 7), (6\ 7\ 10; 3\ 9\ 6), (2\ 4\ 12; 7\ 0\ 5),$
 $(4\ 8\ 12; 1\ 3\ 2), (3\ 6\ 7; 4\ 2\ 10), (1\ 9\ 11; 0\ 7\ 7), (2\ 5\ 10; 8\ 6\ 10),$
 $(1\ 6\ 9\ 11; 10\ 1\ 4\ 3\ 6\ 3), (2\ 4\ 8\ 10; 3\ 0\ 0\ 9\ 9\ 0), (3\ 5\ 7\ 12; 8\ 3\ 5\ 7\ 9\ 2),$
 $(1\ 4\ 7\ 11; 0\ 11\ 3\ 11\ 3\ 4), (2\ 6\ 8\ 12; 7\ 5\ 6\ 10\ 11\ 1), (3\ 5\ 9\ 10; 3\ 5\ 4\ 2\ 1\ 11),$
 $(1\ 5\ 7\ 11; 7\ 0\ 2\ 5\ 7\ 2), (2\ 4\ 8\ 12; 9\ 1\ 4\ 4\ 7\ 3), (3\ 6\ 9\ 10; 9\ 3\ 3\ 6\ 6\ 0),$
 $(2\ 6\ 7\ 12; 5\ 9\ 7\ 4\ 2\ 10), (1\ 4\ 9\ 10; 5\ 9\ 10\ 4\ 5\ 1), (3\ 5\ 8\ 11; 6\ 10\ 8\ 4\ 2\ 10),$
 $(1\ 6\ 9\ 11; 0\ 8\ 9\ 8\ 9\ 1), (3\ 4\ 7\ 12; 1\ 8\ 3\ 7\ 2\ 7), (2\ 5\ 8\ 10; 11\ 11\ 8\ 0\ 9\ 9),$
 $(1\ 5\ 7\ 12; 1\ 2\ 5\ 1\ 4\ 3), (2\ 4\ 9\ 11; 10\ 8\ 8\ 10\ 10\ 0), (3\ 6\ 8\ 10; 0\ 3\ 10\ 3\ 10\ 7),$
 $(1\ 5\ 7\ 11; 8\ 6\ 5\ 10\ 9\ 11), (2\ 6\ 8\ 12; 2\ 9\ 8\ 7\ 6\ 11), (3\ 4\ 9\ 10; 7\ 4\ 8\ 9\ 1\ 4),$
 $(1\ 5\ 9\ 10; 11\ 11\ 1\ 0\ 2\ 2), (2\ 4\ 8\ 11; 4\ 2\ 1\ 10\ 9\ 11), (3\ 6\ 7\ 12; 1\ 9\ 2\ 8\ 1\ 5),$

$(2\ 5\ 9\ 11; 0\ 7\ 6\ 7\ 6\ 11)$, $(3\ 4\ 7\ 10; 5\ 10\ 9\ 5\ 4\ 11)$, $(1\ 6\ 8\ 12; 5\ 9\ 1\ 4\ 8\ 4)$,
 $(1\ 5\ 7\ 10; 6\ 9\ 11\ 3\ 5\ 2)$, $(2\ 6\ 8\ 12; 9\ 3\ 1\ 6\ 4\ 10)$, $(3\ 4\ 9\ 11; 6\ 9\ 6\ 3\ 0\ 9)$,
 $(2\ 5\ 7\ 10; 7\ 11\ 2\ 4\ 7\ 3)$, $(1\ 4\ 9\ 12; 6\ 5\ 6\ 11\ 0\ 1)$, $(3\ 6\ 8\ 11; 7\ 9\ 9\ 2\ 2\ 0)$,
 $(2\ 4\ 9\ 10; 0\ 2\ 10\ 2\ 10\ 8)$, $(1\ 6\ 7\ 12; 2\ 1\ 0\ 11\ 10\ 11)$, $(3\ 5\ 8\ 11; 2\ 8\ 5\ 6\ 3\ 9)$,
 $(2\ 5\ 9\ 12; 2\ 10\ 9\ 8\ 7\ 11)$, $(1\ 6\ 7\ 10; 9\ 3\ 4\ 6\ 7\ 1)$, $(3\ 4\ 8\ 11; 2\ 1\ 7\ 11\ 5\ 6)$,
 $(3\ 5\ 9\ 10; 9\ 2\ 5\ 5\ 8\ 3)$, $(1\ 6\ 8\ 12; 4\ 4\ 11\ 0\ 7\ 7)$, $(2\ 4\ 7\ 11; 6\ 4\ 0\ 10\ 6\ 8)$,
 $(2\ 4\ 7\ 11; 1\ 10\ 3\ 9\ 2\ 5)$, $(1\ 6\ 9\ 10; 8\ 10\ 7\ 2\ 11\ 9)$, $(3\ 5\ 8\ 12; 10\ 7\ 4\ 9\ 6\ 9)$,
 $(3\ 5\ 9\ 12; 7\ 6\ 9\ 11\ 2\ 3)$, $(2\ 6\ 7\ 10; 6\ 3\ 11\ 9\ 5\ 8)$, $(1\ 4\ 8\ 11; 11\ 11\ 0\ 0\ 1\ 1)$,
 $(2\ 5\ 8\ 10; 5\ 6\ 4\ 1\ 11\ 10)$, $(3\ 4\ 9\ 12; 3\ 10\ 7\ 7\ 4\ 9)$, $(1\ 6\ 7\ 11; 11\ 4\ 11\ 5\ 0\ 7)$,
 $(1\ 5\ 8\ 10; 3\ 2\ 6\ 11\ 3\ 4)$, $(3\ 6\ 7\ 12; 6\ 6\ 6\ 0\ 0\ 0)$, $(2\ 4\ 9\ 11; 11\ 5\ 7\ 6\ 8\ 2)$,
 $(1\ 6\ 9\ 12; 1\ 2\ 4\ 1\ 3\ 2)$, $(3\ 4\ 8\ 10; 0\ 2\ 7\ 2\ 7\ 5)$, $(2\ 5\ 7\ 11; 6\ 2\ 5\ 8\ 11\ 3)$,
 $(2\ 6\ 9\ 11; 11\ 6\ 2\ 7\ 3\ 8)$, $(3\ 4\ 7\ 10; 11\ 7\ 11\ 8\ 0\ 4)$, $(1\ 5\ 8\ 12; 0\ 7\ 3\ 7\ 3\ 8)$,
 $(2\ 4\ 7\ 10; 5\ 8\ 1\ 3\ 8\ 5)$, $(1\ 5\ 8\ 12; 10\ 3\ 8\ 5\ 10\ 5)$, $(3\ 6\ 9\ 11; 2\ 7\ 0\ 5\ 10\ 5)$,
 $(2\ 6\ 9\ 10; 1\ 11\ 9\ 10\ 8\ 10)$, $(3\ 5\ 8\ 11; 4\ 0\ 4\ 8\ 0\ 4)$, $(1\ 4\ 7\ 12; 4\ 5\ 2\ 1\ 10\ 9)$,
 $(1\ 6\ 8\ 11; 6\ 5\ 10\ 11\ 4\ 5)$, $(3\ 5\ 7\ 10; 1\ 1\ 1\ 0\ 0\ 0)$, $(2\ 4\ 9\ 12; 2\ 3\ 3\ 1\ 1\ 0)$,
 $(3\ 6\ 7\ 11; 10\ 11\ 11\ 1\ 1\ 0)$, $(2\ 5\ 9\ 12; 3\ 1\ 11\ 10\ 8\ 10)$, $(1\ 4\ 8\ 10; 1\ 6\ 0\ 5\ 11\ 6)$.

Example A.25 A uniform $\{3, 4\}$ -LRGDD₈ of type 3^4 with $r_3 = 12$ and $r_4 = 16$,

$G = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}, \{10, 11, 12\}\}$; each row forms a uniform parallel class:

$(4\ 9\ 11; 7\ 1\ 2)$, $(3\ 6\ 12; 5\ 2\ 5)$, $(2\ 8\ 10; 3\ 3\ 0)$, $(1\ 5\ 7; 3\ 5\ 2)$,
 $(1\ 5\ 12; 5\ 5\ 0)$, $(4\ 8\ 10; 4\ 1\ 5)$, $(3\ 6\ 7; 7\ 2\ 3)$, $(2\ 9\ 11; 3\ 1\ 6)$,
 $(2\ 5\ 8; 2\ 2\ 0)$, $(3\ 4\ 10; 6\ 1\ 3)$, $(6\ 7\ 11; 4\ 6\ 2)$, $(1\ 9\ 12; 2\ 2\ 0)$,
 $(2\ 8\ 12; 5\ 7\ 2)$, $(1\ 6\ 11; 2\ 2\ 0)$, $(4\ 9\ 10; 6\ 5\ 7)$, $(3\ 5\ 7; 0\ 7\ 7)$,
 $(3\ 8\ 10; 0\ 3\ 3)$, $(4\ 7\ 11; 3\ 7\ 4)$, $(2\ 5\ 12; 0\ 3\ 3)$, $(1\ 6\ 9; 6\ 3\ 5)$,
 $(5\ 7\ 11; 6\ 4\ 6)$, $(2\ 9\ 10; 4\ 4\ 0)$, $(1\ 6\ 8; 0\ 7\ 7)$, $(3\ 4\ 12; 0\ 5\ 5)$,
 $(5\ 9\ 10; 4\ 0\ 4)$, $(1\ 6\ 11; 4\ 0\ 4)$, $(3\ 7\ 12; 5\ 6\ 1)$, $(2\ 4\ 8; 6\ 1\ 3)$,
 $(2\ 5\ 11; 7\ 7\ 0)$, $(6\ 8\ 10; 5\ 1\ 4)$, $(3\ 4\ 9; 5\ 2\ 5)$, $(1\ 7\ 12; 4\ 3\ 7)$,
 $(5\ 7\ 12; 0\ 2\ 2)$, $(2\ 4\ 11; 0\ 5\ 5)$, $(3\ 6\ 9; 6\ 7\ 1)$, $(1\ 8\ 10; 3\ 1\ 6)$,
 $(5\ 9\ 12; 5\ 4\ 7)$, $(2\ 6\ 10; 7\ 5\ 6)$, $(3\ 7\ 11; 3\ 3\ 0)$, $(1\ 4\ 8; 1\ 2\ 1)$,
 $(3\ 5\ 10; 1\ 0\ 7)$, $(6\ 8\ 12; 6\ 3\ 5)$, $(2\ 4\ 9; 3\ 6\ 3)$, $(1\ 7\ 11; 6\ 1\ 3)$,
 $(3\ 9\ 11; 3\ 7\ 4)$, $(6\ 8\ 12; 0\ 0\ 0)$, $(1\ 4\ 10; 6\ 5\ 7)$, $(2\ 5\ 7; 4\ 0\ 4)$,
 $(2\ 6\ 8\ 10; 1\ 4\ 6\ 3\ 5\ 2)$, $(1\ 5\ 9\ 11; 6\ 0\ 7\ 2\ 1\ 7)$, $(3\ 4\ 7\ 12; 3\ 1\ 1\ 6\ 6\ 0)$,
 $(2\ 5\ 9\ 11; 1\ 0\ 0\ 7\ 7\ 0)$, $(3\ 4\ 8\ 12; 1\ 7\ 0\ 6\ 7\ 1)$, $(1\ 6\ 7\ 10; 1\ 0\ 0\ 7\ 7\ 0)$,
 $(1\ 4\ 8\ 11; 3\ 0\ 6\ 5\ 3\ 6)$, $(2\ 5\ 7\ 10; 6\ 7\ 2\ 1\ 4\ 3)$, $(3\ 6\ 9\ 12; 0\ 0\ 4\ 0\ 4\ 4)$,
 $(2\ 6\ 7\ 10; 0\ 1\ 0\ 1\ 0\ 7)$, $(1\ 5\ 9\ 12; 1\ 4\ 7\ 3\ 6\ 3)$, $(3\ 4\ 8\ 11; 2\ 1\ 6\ 7\ 4\ 5)$,
 $(1\ 4\ 8\ 12; 4\ 4\ 0\ 0\ 4\ 4)$, $(3\ 5\ 7\ 10; 5\ 0\ 2\ 3\ 5\ 2)$, $(2\ 6\ 9\ 11; 2\ 1\ 4\ 7\ 2\ 3)$,
 $(2\ 6\ 7\ 12; 6\ 6\ 4\ 0\ 6\ 6)$, $(1\ 4\ 9\ 10; 0\ 1\ 6\ 1\ 6\ 5)$, $(3\ 5\ 8\ 11; 2\ 4\ 5\ 2\ 3\ 1)$,
 $(2\ 5\ 9\ 12; 5\ 5\ 6\ 0\ 1\ 1)$, $(3\ 6\ 8\ 11; 1\ 2\ 4\ 1\ 3\ 2)$, $(1\ 4\ 7\ 10; 7\ 7\ 3\ 0\ 4\ 4)$,
 $(2\ 6\ 9\ 11; 4\ 2\ 3\ 6\ 7\ 1)$, $(3\ 4\ 7\ 10; 4\ 6\ 4\ 2\ 0\ 6)$, $(1\ 5\ 8\ 12; 7\ 5\ 4\ 6\ 5\ 7)$,
 $(3\ 6\ 9\ 10; 2\ 5\ 6\ 3\ 4\ 1)$, $(2\ 4\ 7\ 11; 2\ 3\ 2\ 1\ 0\ 7)$, $(1\ 5\ 8\ 12; 2\ 6\ 1\ 4\ 7\ 3)$,
 $(3\ 5\ 8\ 11; 4\ 3\ 2\ 7\ 6\ 7)$, $(1\ 6\ 7\ 10; 7\ 1\ 2\ 2\ 3\ 1)$, $(2\ 4\ 9\ 12; 7\ 7\ 1\ 0\ 2\ 2)$,
 $(3\ 6\ 9\ 12; 4\ 6\ 3\ 2\ 7\ 5)$, $(2\ 5\ 8\ 10; 3\ 0\ 1\ 5\ 6\ 1)$, $(1\ 4\ 7\ 11; 5\ 2\ 3\ 5\ 6\ 1)$,
 $(3\ 5\ 7\ 11; 7\ 4\ 1\ 5\ 2\ 5)$, $(1\ 4\ 9\ 10; 2\ 6\ 4\ 4\ 2\ 6)$, $(2\ 6\ 8\ 12; 3\ 7\ 5\ 4\ 2\ 6)$,
 $(2\ 4\ 8\ 11; 4\ 6\ 6\ 2\ 2\ 0)$, $(1\ 6\ 7\ 12; 5\ 3\ 6\ 6\ 1\ 3)$, $(3\ 5\ 9\ 10; 6\ 4\ 7\ 6\ 1\ 3)$,
 $(3\ 4\ 9\ 12; 7\ 1\ 7\ 2\ 0\ 6)$, $(2\ 6\ 7\ 10; 5\ 2\ 7\ 5\ 2\ 5)$, $(1\ 5\ 8\ 11; 0\ 1\ 5\ 1\ 5\ 4)$,
 $(3\ 6\ 8\ 11; 3\ 5\ 0\ 2\ 5\ 3)$, $(2\ 4\ 7\ 12; 1\ 5\ 2\ 4\ 1\ 5)$, $(1\ 5\ 9\ 10; 4\ 5\ 7\ 1\ 3\ 2)$,
 $(3\ 5\ 8\ 10; 3\ 6\ 5\ 3\ 2\ 7)$, $(2\ 4\ 7\ 12; 5\ 4\ 0\ 7\ 3\ 4)$, $(1\ 6\ 9\ 11; 3\ 7\ 4\ 4\ 1\ 5)$.